Available online at www.sciencedirect.com
ScienceDirect
Structural Safety 30 (2008) 517–532
STRUCTURAL SAFETY
www.elsevier.com/locate/strusafe

Reliability sensitivity method by line sampling

Zhenzhou Lu [a,*], Shufang Song [a], Zhufeng Yue [b], Jian Wang [c]

[a] School of Aeronautics, Northwestern Polytechnical University, Xian 710072, China
[b] School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xian 710072, China
[c] School of Mechanical and Aerospace Engineering, Queen's University Belfast, Ashby Building, Stranmillis Road, Belfast BT9 5AH, Northern Ireland, United Kingdom

Received 26 August 2006; received in revised form 1 September 2007; accepted 1 October 2007
Available online 26 November 2007

Abstract

Reliability sensitivity refers to the derivative of the failure probability with respect to the distribution parameter of basic random variable. Conventionally, this requires repetitive evaluations of the failure probability for different distribution parameters, which is a direct but computationally expensive task. An efficient simulation algorithm is presented to perform reliability sensitivity analysis using the line sampling technique, which gives a good failure probability evaluation for high-dimensional problems and still presents a comparative one for low-dimensional problems. On the basis of the line sampling procedure for failure probability analysis, the concept and implementation are presented for reliability sensitivity. It is shown that the desired information about reliability sensitivity can be obtained by a very limited increase of computation effort based on the failure probability analysis by the line sampling technique. The presented reliability sensitivity algorithm is more efficient than the one based on the direct Monte Carlo technique, especially for cases where the failure probability is low and the number of random variables is large, which is illustrated by several examples. Additionally, limitations of the line sampling based reliability sensitivity method are demonstrated by a numerical example as well.

Keywords: Reliability; Sensitivity; Line sampling; Failure probability

1. Introduction

The reliability sensitivity refers to the partial derivative of the failure probability with respect to the distribution parameter of basic random variable. It can rank the distribution parameters of the design variables and guide the reliability based design, therefore it is important to develop an efficient method for assessing reliability sensitivity. This paper provides a novel reliability sensitivity method that is based on the line sampling which is a new technique for evaluating the reliability of the structure with small failure probability and many random basic variables.

* Corresponding author. Tel.: +86 29 88460480.
E-mail address: zhenzhoulu@nwpu.edu.cn (Z. Lu).

0167-4730/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.strusafe.2007.10.001

Although the reliability sensitivity analysis and the reliability analysis serve different aims, in practice, the implementation of the reliability analysis and that of the reliability sensitivity analysis are very closely connected on both a conceptual and a computational level [1]. Thus the available reliability sensitivity methods are mostly based on the corresponding reliability analysis methods. After brief review of several reliability sensitivity methods, why the line sampling method is used to construct the novel reliability sensitivity method is explained.

Ditlevsen and Madsen [2] presented an expression based on the first-order reliability method to evaluate reliability sensitivity for structural system with linear limit state and normal random variables. An accurate reliability sensitivity result can be obtained by Ditlevsen's method in the case of the limit state function being linear and the random variables being normal distribution. However, for the non-linear limit state function, this method can only give an approximate result with an unknown precision. As a development of Ditlevsen's method, Melchers and Ahammed [3] proposed a fast approximate method for parameter sensitivity estimation in Monte Carlo structural reliability. When the structural reliability is estimated by the Monte Carlo simulation, Melchers' method could provide the reliability sensitivity with a very limited increase in computation cost. However, the Monte Carlo simulation is time-consuming for the small failure probability estimation with high dimension of basic random variables, especially for a large scale structure with its response controlled by the finite element method.

Combining the most probable point system simulation (MPPSS) and the sampling based reliability sensitivity method [4,5], Sues and Cesare [6] proposed a simple reliability sensitivity method for multiple failure modes by constructing the linearization approximations of the non-linear performance functions. Obviously, the linearization of the non-linear performance function will lead to a loss in precision in the reliability evaluation and the reliability sensitivity evaluation.

Wu [4,5] presented a reliability sensitivity method based on the cumulative distribution function (CDF) of the structural response variable, wherein the sampling based method can be used to compute the reliability sensitivity. Although the variance reduced sampling method (e.g. importance sampling) is more efficient than Monte Carlo sampling, the computation cost is still expensive. Especially for the high dimensional problems, the precision of the importance sampling-based reliability sensitivity analysis is problematic, which is the same as the precision problem of the importance sampling-based reliability analysis for high dimensionality.

Au [7] presented a method of reliability-based design sensitivity analysis by efficient simulation. Au's approach is based on consideration of an 'augmented reliability problem' where the design parameters are artificially considered as uncertain. The efficient simulation of the approach is a subset simulation on the basis of Markov Chain Monte Carlo (MCMC). Although MCMC simulation is a class of powerful simulation methodologies [8–13], the required computational effort in the reliability sensitivity analysis grows exponentially with the number of design parameters whose sensitivity is to be studied, therefore it is not applicable to a large number of design parameters.

The moment method was presented to analyze the reliabilities of both series structure systems and non-series structure systems by Zhao and Ono in Refs. [14–17]. Since the moment based reliability method requires neither the computation of derivatives nor the determination of the design point and the computation of mutual correlations among failure modes, the procedure should be computationally effective for the reliability assessment [16]. Based on the moment method for reliability analysis, the authors of this paper established a quantitative and simple method for reliability sensitivity analysis. The advantages and the disadvantages shown in Ref. [17] of the moments based reliability analysis are all propagated to the moments based reliability sensitivity analysis simultaneously. The major disadvantage of the moment based reliability sensitivity method involves inapplicability to the high dimension problems, because the required computational effort grows fast with the number of basic random variables.

Line sampling and subset simulation are declared as effective methodologies for reliability analysis in the high dimensionality [7–13,18]. The line sampling does not require any approximating surrogate of the limit state surface and therefore combines the property of robustness with accuracy [12]. Moreover, multiple failure domains can be treated quite simply and straightforwardly. Therefore the line sampling technique is used as a basis of the reliability sensitivity method presented in this paper. By use of the line sampling technique and the Ditlevsen's reliability sensitivity method, the concept and the implementation of the presented line sampling based reliability sensitivity are explained in the following sections.