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Transforming reliability limit-state constraints into deterministic limit-state constraints

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Abstract

This research proposes an efficient methodology that can transform a reliability limit-state constraint into a deterministic limit-state constraint. This new method may have broad applications in reliability-based design and optimization. The method is based on subset simulation and is applicable to general systems, e.g. linear or nonlinear systems and static and dynamical systems with high-dimensional uncertainties. Once the reliability constraint is transformed into a deterministic one, it is no longer necessary to conduct reliability analyses to verify the former. This can potentially save lots of computational time during the process of reliability-based design and optimization. The main idea of the new method is the introduction of the limit-state factor and nominal limit-state function, from which a deterministic nominal limit-state constraint is established. We propose two methods of establishing this nominal constraint, and they are verified with several simulated examples, showing that the established nominal constraint is indeed a satisfactory approximation of the target reliability constraint. Finally, we propose a simplified procedure requiring only two reliability analyses that can effectively transform reliability constraints into deterministic nominal constraints.

Keywords: Reliability analysis; Reliability-based design; Reliability-based optimization; Subset simulation

1. Introduction

Uncertainties are abundant in civil engineering systems, and reliability analysis [1–7] is the main tool of quantifying these uncertainties. In fact, reliability analysis plays major roles in several relevant research areas, including reliability-based optimization and design [8–11], performance-based engineering [12,13] and life-cycle engineering [14–17].

In practice, we are usually required to conduct reliability analysis under various design settings. Using reliability-based optimization as an example, let us consider the following optimization problem:

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$$\min_{\theta} c_0(\theta) \quad \text{s.t.} \quad P(F|\theta) \leqslant P_F^* \quad c_i(\theta) \leqslant 0 \quad i = 1, \ldots, m, \tag{1}$$

where F is the failure event of the target system; θ contains the design variables; $P(F|\theta)$ is the failure probability given θ; P_F^* is the target failure probability; $c_0(\theta)$ denotes the objective function; $\{c_i(\theta) \leqslant 0 : i = 1, \ldots, m\}$ are the deterministic constraints. If the reliability constraint $P(F|\theta) \leqslant P_F^*$ is transformed into a deterministic constraint, (1) can then be transformed into an ordinary optimization problem. Otherwise, reliability-based optimization can be quite time consuming because large amount of reliability analyses may be required to ensure the reliability constraint is satisfied during the optimization algorithm.

Transforming a reliability constraint $P(F|\theta) \leqslant P_F^*$ into a deterministic constraint is not a trivial task. In recent years, much research effort [18–20] is taken to achieve this goal by using the first-order reliability method (FORM). The fundamental assumption with FORM is that the limit-state function is linear; therefore, it is not applicable to highly nonlinear problems. Furthermore, FORM is not efficient for problems with high-dimensional uncertainties. Transforming reliability constraints into deterministic constraints for general nonlinear problems and problems with high-dimensional uncertainties remains a challenging research topic.

In this paper, a new methodology based on subset simulation [4,21–23] is developed to achieve the transformation. With the simulated examples, it is shown that this new method is efficient and effective in transforming reliability constraints into deterministic constraints for general problems with high-dimensional uncertainties, which can encompass uncertain system parameters and excitation uncertainties. At the end, a simplified method that only requires two reliability analyses of achieving the transformation is proposed. This new framework may be valuable to reliability-based design and optimization.

The structure of this paper is as following: In Section 2, the concept of limit-state factors, nominal limit-state functions and nominal limit-state constraints is introduced, and a theorem is developed for the transformation from reliability constraints to deterministic nominal constraints. In Section 3, the significance of limit-state factors is further studied. Two ways of approximating the factors are described, and the detailed algorithms of estimating them are discussed in Section 4. These algorithms are based on subset simulation. In Section 5, three simulated examples are used to demonstrate the new methodology. Last but not least, Section 6 contains discussions of the new approach and proposes a simplified method that only requires two reliability analyses to achieve the transformation.

2. Problem definition

Let Z and θ be uncertain variables and design variables of the target system. Also let D be the allowable design region in the θ space. Let F denotes the failure event: $F \equiv \{R[Z,\theta] > 1\}$, where $R[Z,\theta]$ is called the limit-state function. Throughout the paper, it is assumed that $R[Z,\theta]$ is positive. Consider the following reliability constraints:

$$P(F|\theta) = P(R[Z,\theta] > 1|\theta) = \int p(Z|\theta) \cdot I_F(Z,\theta)\, dZ \leqslant P_F^* \quad \forall \theta \in D, \tag{2}$$

where $p(Z|\theta)$ is the probability density function (PDF) of Z given θ; $I_F(Z,\theta)$ is the indicator function of the failure event F, i.e. if $R[Z,\theta] > 1$, $I_F(Z,\theta) = 1$; otherwise, $I_F(Z,\theta) = 0$.

The constraint (2) is often encountered in reliability-based design and optimization. Nonetheless, the implementation of this constraint can be troublesome in practice. Using reliability-based optimization as an example, before the final design solution is obtained, numerous reliability analyses may be required to verify if the reliability constraint is met during the optimization search. This paper proposes a new method of transforming the constraint (2) into a deterministic constraint. Once this transformation is done, verifying the constraint no longer requires repetitive reliability analyses.

Before the method is introduced, let us consider the following theorem. Assume that given θ, the PDF of $R[Z,\theta]$ is non-zero everywhere in the Z space. The situation that this assumption is violated will be addressed later. If a $\eta(\theta)$ function can be found such that

$$P(R[Z,\theta] - \eta(\theta)R_n(\theta) > 0|\theta) = P_F^* \quad \forall \theta \in D, \tag{3}$$