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Interactively Induced Localization in Thin-walled I-section Struts Buckling About the Strong Axis

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ABSTRACT

A variational model describing the behaviour of a thin-walled I-section strut suffering from local–global buckling mode interaction is presented where global (Euler) buckling about the strong axis is the critical mode. A system of differential and integral equations is derived that describe the equilibrium states from variational principles and are solved numerically using the continuation and bifurcation software Auto-07p for the perfect case. Initially stress relieved out-of-straightness imperfections are subsequently introduced and the nonlinear response is modelled. The modelled interaction is between the critical global buckling mode about the strong axis and local buckling in the flange and web simultaneously, where the flange–web joint is assumed to be free to rotate as a rigid body. The initial eigenmode is shown to be destabilized at a secondary bifurcation where interactive buckling is triggered. A progressive change in the buckling mode is then observed, initially with local buckling localizing at the mid-span of the compression flange, which also triggers sympathetic local buckling in the web. The results from the analytical model have been validated using the commercial finite element (FE) software Abaqus with good comparisons presented for the initial post-buckling behaviour. The strut also exhibits sensitivity to initial out-of-straightness imperfections, with a notable decrease in the ultimate load as the imperfection size increases. The ultimate loads for a range of imperfection amplitudes are found using both analytical models and FE analysis, with very good correlation observed.

1. Introduction

Open section, thin-walled columns and struts are well known to be susceptible to buckling in a variety of different modes. Moreover, where local and global instability modes have similar buckling loads, the structure is known to be susceptible to exhibiting nonlinear modal interactions [1]. The interactive post-buckling response of these structures can introduce significantly less stable behaviour than if either mode were to be triggered independently, potentially reducing the load carrying capacity considerably. Extensive numerical and experimental studies have been previously conducted on thin-walled struts and columns, with evidence of interaction between global, local and distortional modes of buckling [2,3,4,5,6]. Such systems have also been shown previously to be sensitive to imperfections, which can further decrease the load capacity [7,8,9]. However, since thin-walled struts are highly mass-efficient, with large load capacity to self-weight ratios, these components are used extensively in industry, particularly in the civil, maritime and aeronautical engineering sectors [10]. It is therefore essential that further understanding of the behaviour of these types of structures is developed since they offer significant practical advantages even though they can suffer from complex and potentially dangerous instabilities that can be amplified by geometric imperfections.

The current work presents an analytical model of a thin-walled, linear elastic, doubly symmetric I-section strut of uniform thickness under pure compression. In recent work [11], a similar structure was studied where geometries dictated global buckling about the weak axis to be the critical mode, which then triggered local buckling in both the flange and the web, as is often observed in practice. The current study focuses on the case where weak axis buckling is restrained with global buckling about the strong axis becoming critical. This is often seen in applications where the system is braced in order to increase the load carrying capacity by reducing the buckling length in the weak axis. The strong axis is often left unbraced and thus may become critical. Given the geometric arrangement, this may naturally push the global and local modes of buckling to be triggered at more similar loading levels. Moreover, with the modern trend of using higher strength materials, in particular steels [12,13], elastic behaviour has regained an increased practical significance.

The model is formulated using variational principles in conjunction with the Rayleigh–Ritz method using a series of displacement functions and generalized coordinates resulting in a system of nonlinear ordinary differential and integral equations. Initially, the perfect case model is analysed and the equations are solved numerically in the continuation and bifurcation software Auto-07p [14]. A strut with identical material

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and geometric properties is also analysed in the commercial finite element (FE) software ABAQUS [15] for comparison and validation purposes. The imperfect case model is then studied, where an initial out-of-straightness is introduced in the major axis of the strut. The equations are solved numerically in a similar manner and the ultimate loads are compared to an FE model formulated using the same magnitude and shape of the initial global buckling deflection.

For the perfect strut, it is found that with the selected geometries, strong axis global buckling is triggered as the initial eigenmode, resulting in a neutrally stable path that is subsequently destabilized at a secondary bifurcation point where local buckling in the more compressed flange is also triggered. Since the flange–web joints are modelled as being free to rotate as a rigid body, sympathetic local buckling is also triggered in the web, a response that is known from recent work on stiffened plates [8]. A nonlinear, interactive post-buckling path is observed in the current study; behaviour that has also been found in other thin-walled components [4,16,17]. The analytical model shows a good comparison with the FE model, particularly close to the point of secondary instability. The imperfect strut response is observed to have a similar post-buckling path to the perfect case; however, it is found to be sensitive to initial out-of-straightness imperfections with a decreasing observed ultimate load as initial deflections increase. It is also found that the ultimate load becomes less sensitive to initial global imperfections as they increase in magnitude. Similar trends and behaviours exhibited in the analytical model have also been observed in experimental studies [5,16,17], indicating that the fundamental physics of the system has been successfully captured using the analytical approach.

2. Analytical model

The elevation and cross-section of the strut under consideration are shown in Fig. 1.

The strut is assumed to be restrained from buckling globally about the weak axis, which is commonly seen in practice where restraints or bracing members are utilized to constrain the buckling length thereby preventing global instability in that direction. In such cases, given sufficient bracing, when instability occurs, it is therefore the relatively unbraced strong axis global mode that is triggered.

The analytical model formulation begins by defining the functions used to describe the global and local buckling shapes. The global mode can be decomposed into two components, which are defined as the 'sway' and 'tilt' components, as used successfully in previous work [18,19]. The two components are shown in Fig. 2, the combination of which allows the development of shear strains within the cross-section, which is a feature of Timoshenko beam theory and has been shown to be a key element for capturing mode interaction successfully in analytical studies [20]. In previous work related to the buckling of I-sections, it has been assumed that the web is rigid and therefore did not deform locally during the post-buckling process [17,21]. However, when the strut buckles globally about the strong axis, inclusion of local web buckling is of paramount importance for modelling interactive behaviour in the system since it provides the only significant source of the terms in the governing equations that exhibit an explicit interaction between the local and global instability modes. Moreover, if the strut is assumed to be 'thin-walled', there would be no significant through-thickness shear strains developed within the flanges.

The sway and tilt kinematic components shown are defined as W and θ respectively:

$$W(z) = q_s L \sin\left(\frac{\pi z}{L}\right), \quad \theta(z) = q_t \pi \cos\left(\frac{\pi z}{L}\right), \tag{1}$$

where q_s and q_t are the respective generalized coordinates for the sway and tilt modes. In addition to the global sway and tilt modes, the initial stress relieved out-of-straightness imperfections, also shown in Fig. 2, are introduced; the corresponding functions W_0 and θ_0 are written as:

$$W_0(z) = q_{s0} L \sin\left(\frac{\pi z}{L}\right), \quad \theta_0(z) = q_{t0} \pi \cos\left(\frac{\pi z}{L}\right), \tag{2}$$

where q_{s0} and q_{t0} are the amplitudes of the global out-of-straightness imperfections.

There are four local buckling displacement components to be defined, the in-plane displacements u_{fl} and u_{wl} of the flange and web respectively, as well as the out-of-plane displacements w_{fl} and w_{wl}, again of the flange and web respectively. The local transverse deflection in the x-direction v is assumed to be small and is thus neglected [22]. Fig. 3 shows the local buckling mode deflections, which are defined as:

$$u_{fl}(x,z) = u_f(z), \quad u_{wl}(y,z) = -\left(\frac{y}{h}\right) u_w(z), \tag{3}$$

$$w_{fl}(x,z) = f(x) w_f(z), \quad w_{wl}(y,z) = g(y) w_w(z), \tag{4}$$

for the in-plane flange and web deflections and the out-of-plane flange and web deflections respectively. As previously mentioned, Timoshenko beam theory is being used throughout the current formulation, thus for the in-plane modes, a linear function in y and a constant in x for the web and flange respectively are selected, such that they fulfil the constraint that plane sections remain plane while bending. Functions $f(x)$ and $g(y)$ define the deflected shapes of the flange in the x-axis and of the web in the y-axis respectively. The functions $f(x)$ and $g(y)$ are selected such that they satisfy the boundary conditions for each separate element while also giving a good representation of the deflected shape of the element.

In the current work, where strong axis global buckling is the critical mode, the flange under most compression can be modelled as being subject approximately to a uniform compression across its entire breadth, particularly before and immediately after global buckling is

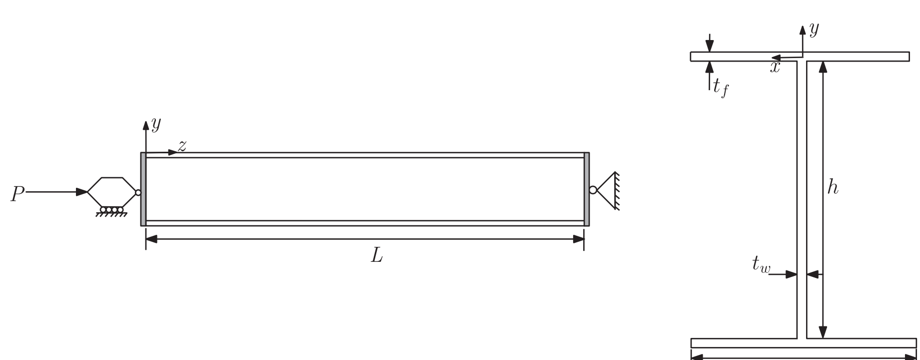

Fig. 1. An I-section strut under axial loading P, elevation (left) and cross-section (right). The ends are simply supported and a rigid end plate transfers the load equally to the flanges.