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Ultimate capacity of structural steel cross-sections under compression, bending and combined loading

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ABSTRACT

The Continuous Strength Method (CSM) is a strain based structural steel design approach which allows for the beneficial influence of strain hardening. The method has been previously developed for predicting compression and bending resistances in isolation. This paper describes extension of the method to enable the prediction of the ultimate cross-section resistance of I-sections and box sections under combined loading. At the core of the method is a base curve, which relates the deformation capacity of a cross-section to its cross-section slenderness. Deformation capacity is defined as the ratio of the maximum strain that a cross-section can endure relative to its yield strain. Knowing this limiting strain and assuming plane sections remain plane, the resistance of a cross-section to combinations of axial load and bending moments can be calculated, by integrating the stresses arising from a suitable strain hardening material model over the area of the cross-section. By considering a range of combinations of applied actions, analytical expressions and numerically derived interaction surfaces have been produced, which were then rationalised into simple expressions for use in design. The resulting CSM design predictions for box sections and I-sections have been compared with existing test data, and shown to give additional capacity over current design approaches and a reduction in scatter of the predictions.

1. Introduction

Design rules for structural steel cross-sections often include simplifications that allow quick and conservative estimates of capacity (e.g. the ability to withstand combinations of axial forces, shear forces and bending moments) to be obtained. Some of these simplifications are at the material level, where structural steel is typically assumed to have an elastic–plastic or rigid–plastic stress–strain ($\sigma - \epsilon$) response, some are based on equations limited by elastic conditions, while other approximations involve the grouping of similar behaviour, as in the case of cross-section classification for the treatment of local buckling. Although generally conservative, these approximations are widely used in modern design codes, such as EN 1993-1-1 [1], with the consequence that material may not be fully utilised and varying reliability levels are achieved for different design scenarios.

At the ultimate limit state, a cross-section subjected to flexure is typically designed on the basis of its plastic ($M_{pl} = W_{pl}f_y$) or elastic moment capacity ($M_{el} = W_{el}f_y$), where W_{pl} and W_{el} are the plastic and elastic section moduli and f_y is the material yield stress. The choice between the two is based on the susceptibility of the cross-section to local buckling, which is assessed by considering the width-to-thickness ratios of the elements that make up the cross-section through a process known as cross-section classification. For slender cross-sections, where local buckling occurs prior to the initiation of yielding, reduced moment capacities are assigned. This approach generally results in a step from M_{el} to M_{pl} at a particular slenderness limit, as is the case in EN 1993-1-1. This has led to the proposal of various elastic–plastic moment transitions to eliminate the discontinuity. The simplest approach is to have a linear transition between M_{el} and M_{pl}, as used in BS 5950-1 [2]. This approach was proposed by Greiner et al. [3] for inclusion in EN 1993-1-1, and also extended to the case of combined loading. A parabolic transition was proposed by Juhás [4] and also by Shifferaw and Schafer [5], with both considering the ratio of maximum strain to yield strain in their derivation. Under combined loading, cross-section capacities are generally assessed through interaction curves that are anchored at the end points to the cross-section resistances under the three individual load cases of axial load, major axis bending and minor axis bending.

Various models have also been proposed for allowing moment capacities greater than the plastic moment. Kemp et al. [6] developed a bi-linear moment–curvature relationship based on bending tests of hot-rolled I-sections, while Byfield and Nethercot [7] put forward two methods for incorporating strain hardening into the design of I-section beams, which involved the material stress at a strain of 1.5%. The Continuous Strength Method aims to harmonise different aspects of structural steel design under one deformation based approach, and to offer more consistent and continuous resistance functions which can also account for strain hardening. At the core of the CSM is a base curve, which establishes a relationship between the deformation capacity of a cross-section and its cross-section slenderness. From the base

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curve, a maximum allowable strain is determined which defines the strain distribution throughout the cross-section. Once the strain distribution is known, the stresses follow via a chosen material model. The cross-section capacity is then obtained by integrating the stresses throughout the cross-section for the appropriate combination of applied loading. To date, this method has been established for cross-sections under axial load and bending in isolation (Gardner [8], Gardner et al. [9] and Afshan and Gardner [10]). The present paper extends the method to all combinations of axial load and bending moments for box sections and I-sections.

2. Components of the CSM

The Continuous Strength Method (CSM) is a deformation (strain) based design method with two key components. The first component is a material model that allows for the influence of strain hardening; this is described in Section 2.1. The second component is a base curve, which defines the maximum strain ϵ_{csm} that a cross-section can endure, as a function of the cross-section slenderness. Development of the base curve, utilising both compression and bending test data, is described in Sections 2.2 to 2.4.

2.1. Material model

The stress–strain ($\sigma - \epsilon$) response of structural steel can differ depending on the material grade and how the material has been manufactured, subsequently mechanically worked, and ultimately tested. Hot-rolling (Fig. 1a) or cold-forming (Fig. 1b) can affect the material behaviour by altering the distinctiveness of the yield point, the length of the yield plateau, and the magnitude of the strain hardening slope. Variation in material properties around structural cross-sections is also possible, such as in the case of cold-formed sections, where higher strength but lower ductility are typically found in the corner regions. Given that the stress–strain response can vary markedly, it is important to utilise a material model that can represent adequately the range of characteristic material curves.

In Fig. 1, E is the Young's modulus, f_y and f_u are the yield and ultimate tensile stresses, $\epsilon_y = f_y/E$ and ϵ_u are the strains at the yield and ultimate stress, $\sigma_{0.2}$ and $\epsilon_{t,0.2}$ are the 0.2% offset proof stress and corresponding strain, E_{sh} is the strain hardening slope and f_{csm} and ϵ_{csm} are the CSM limiting stress and strain. Traditionally a bi-linear, elastic–perfectly plastic material model is used to model structural steel, with the key advantage of being very simple to analyse, but with the potential disadvantage of being overly conservative since no post-yield strain hardening is accounted for. Alternatives to the elastic–perfectly plastic model include bi-linear (elastic–linear hardening), tri-linear, power, Ramberg and Osgood [11] and other stress–strain models. In principle, any material law can be used in conjunction with the deformation based CSM. The proposed material model (Fig. 1c) is an elastic–linear hardening relationship, which consists of an initial linear region with Young's modulus E defining stresses up to the yield stress, followed by a strain hardening region, described by an appropriate strain hardening modulus E_{sh} for the material. A maximum limiting strain is also set at 15 times the yield strain ($\epsilon_{csm}/\epsilon_y = 15$), a value which corresponds to the material ductility requirements given in Clause 3.2.2(1) of EN 1993-1-1. This material model gives the following stress–strain relationship:

$$\sigma = \begin{cases} E\epsilon & \epsilon \leq \epsilon_y \\ f_y + E_{sh}\left(\epsilon - \epsilon_y\right) & \epsilon_y < \epsilon \leq \epsilon_{csm} \end{cases}. \tag{1}$$

The key characteristic to be defined in the adopted material model is the strain hardening modulus E_{sh}, which should be representative of the whole cross-section. A value of $E_{sh}/E = 1/100$ is adopted for structural steel following the recommendation given in Annex C of EN 1993-1-5 [12], though a value of $E_{sh} = 0$ may be required for some hot-rolled cross-sections that exhibit an extended yield plateau.

2.2. Cross-section slenderness

Local plate buckling may initiate before or after the onset of material yielding, with the key determining geometric factor being the relative width-to-thickness ratios of the plate elements that make up the cross-section. Plate slenderness is commonly defined in the non-dimensional form of Eqn. (2):

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{2}$$

where σ_{cr} is the elastic buckling stress, which is influenced by the boundary and loading conditions of the plate. The plate slenderness values of all the elements that make up the cross-section are evaluated, with the critical and governing element determined as that with the highest value of $\bar{\lambda}_p$. This approach ignores element interaction, but is commonly used in design codes, including EN 1993-1-1 and EN 1993-1-5.

Since basing the cross-section slenderness upon the most slender constituent plate element does not consider the connectivity between the plates, Seif and Schafer [13] formulated expressions for predicting

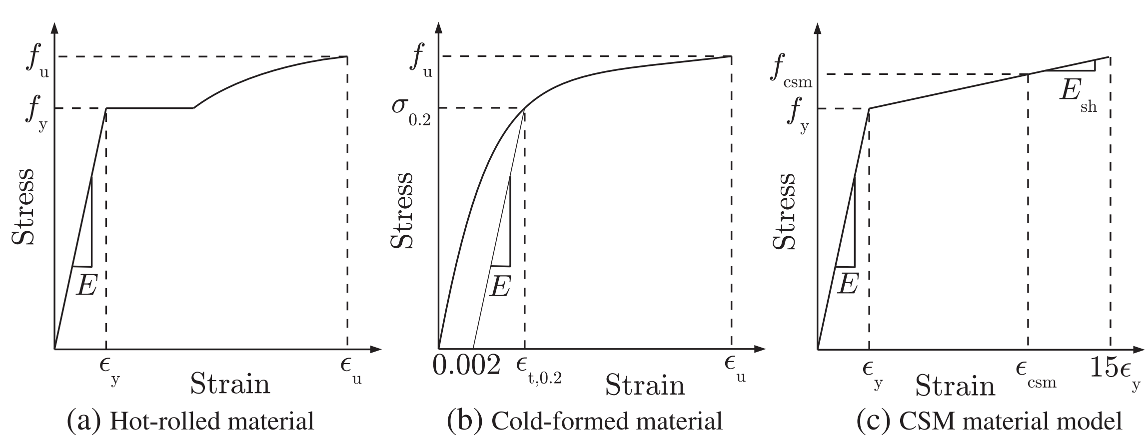

Fig. 1. Schematic stress–strain curves (a) for hot-rolled material, (b) cold-formed material and (c) the CSM material model.