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The deformation mechanism analysis of a circular tube under free inversion

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ABSTRACT

As a popularly adopted design of thin-walled energy absorber, a circular tube under free inversion has been studied theoretically since 1960s However, in all the existing models, the profile of inversion is assumed to be a semi-circle, i.e., the curvature in the region of inversion is a constant. In this study, from equilibrium of a representative element, the curvature is found to be varying, due to bending moment distribution. The conclusion of varying curvature is verified by the corresponding finite element simulations.

Then the deformation mechanism of a tube under free inversion is theoretically investigated by establishing a curvature varying model. Considering the interaction of bending and tension, the steady inversion load and knuckle radius predicted by current model agree well with those obtained from FE simulations as well as the experimental data published in literature.

1. Introduction

Thin-walled metal tube is a kind of commonly used energy absorber. For a circular tube under axial compression, the energy could be dissipated by progress buckling [1,2], global buckling [3], inversion [4], expansion or even splitting [5,6]. Among the above mentioned different collapse modes, inversion possesses a long steady stroke, during which the inversion force remains almost constant, so it is regarded as an ideal energy absorber.

There are two types of tube inversions: free inversion, and inversion with a die. Besides, a circular tube can be inverted externally or internally. In the present paper, only the externally free inverting is studied. As shown in Fig. 1, a metal tube is fixed at one end and compressed at the other end, then it is inverted freely and formed into a co-axial cylinder outside the original one.

The theoretical studies of free inversion are aimed at obtaining a steady inversion load P and a natural knuckle radius b [4,7–11], which depend on tube geometries and material properties, but not the initially induced boundary condition. Most theoretical analysis are based on two-dimensional models. Guist and Marble [7] proposed the original two-dimensional model and predicted the steady knuckle radius and inversion force, where rigid, perfectly plastic material idealization (R-PP) was employed. In their model, curvature, tube length and thickness in the inversion region were all assumed to be constant during the inverting process; which is in conflict with the material volume conservation. Compared to experimental data, their prediction on knuckle radius is almost doubled, and inversion force is about 20–30% lower. After that, researchers keep on trying to improve this model. For example, Kinkead [9] considered the additional circumferential bending energy dissipation, which improved the prediction of steady inversion load P; that is, deviation was reduced to the range of -10% to 10%. But the prediction on knuckle radius b was not improved. Calladine [12] modified the positon of plastic hinges; and Reddy [8] changed the material property to rigid, linearly strain-hardening material; Qiu and He [11] modified velocity at the exit point. All above attempts have improved the steady inversion force or knuckle radius to some extent. However, there are still some deviations between theoretical predictions and finite element simulations.

A two-stage assumption was proposed by Colokoglu and Reddy [10], in which the inversion region was divided into two parts: in first part, tube wall thickness is assumed to be constant; while in second part, material velocity is assumed to be constant. The knuckle radius given by this model [10] is almost the same as that of Reddy's model [8], only 2% higher. Inherited this two-stage assumption [10], Qiu et al. [4] proposed a three-dimensional model considering the strain variation along wall thickness direction, in which volume conservation and Von Mises yield criteria were adopted. This three-dimensional model [4] did not improve the

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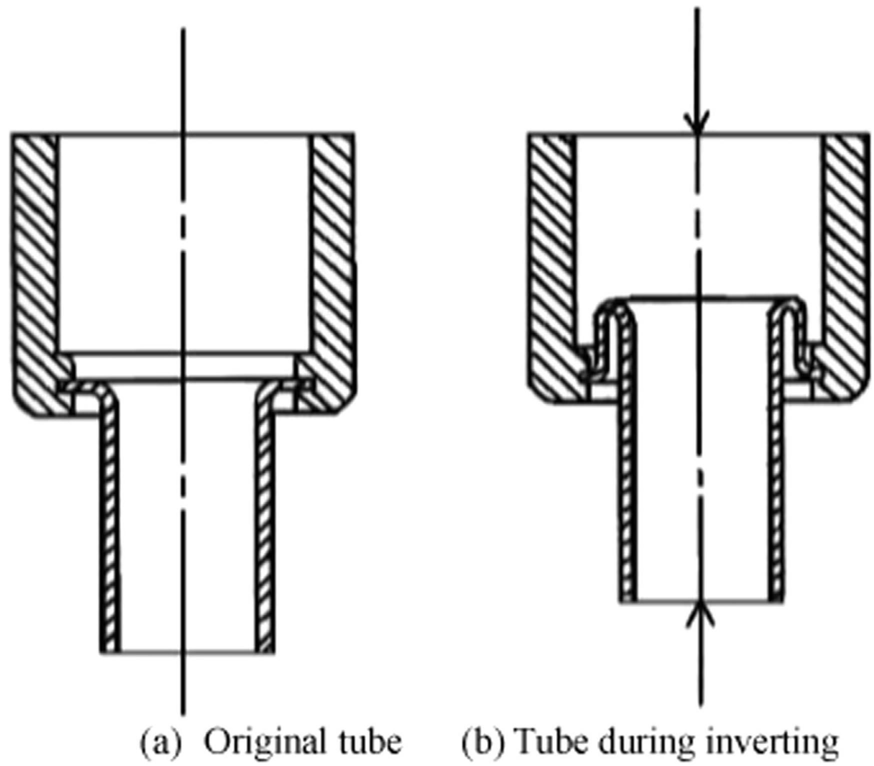

Fig. 1. The external free inversion of a circular tube under axial compression [4].

predictions accuracy on knuckle radius, but did improve the prediction of steady inversion load.

By comparing the initial state and final state, a two-dimensional deformation theory model was then proposed by Yu [13]. In this study, three different deformation mechanisms were analyzed, and among them the results based on a two-stage assumption about tube thickness [10] agree well with those obtained from FE simulations as well as the experimental data.

In summary, the existing theoretical models of free inversion tube are all based on the assumption of inversion region is a semi-circle [4,7–13]. Therefore, an interesting question arises; that is, *in the deformation process of a free inverting tube, is the inversion region really a semi-circle with constant curvature*?

In latest research on tube inversion over a die, the curvature change in inversion region was observed. For example, behavior of the free deformation under die-less constraint was studied by He et al. [14] by employing FE method. It was found that before reaching the steady-state, the curvature at each point in the free deformation zone were changing continuously with time. Employing an energy method, the curling behavior of inverted tubes was analyzed by Leu [15], which led to the determination of inversion force and curling radius. In the case of a quarter circle die for tube inversion, although tube is intimate contact with die, it will curl up when becomes free with a curvature different from that of the fillet die radius. Compared to the constant curvature models [16,17], energy dissipation due to curvature change was considered in Ref. [15].

Inspired by studies of tube inversion over a die [14,15], the deformation mechanism of inversion region of a free inversion tube will be revisited in this study. Firstly, the variation of thickness and curvature in reversion region will be checked. Then based on previous work, a two-dimensional theoretical model is proposed in order to analyze the free inverting metal tube. In Section 2, the two-dimensional deformation theory model proposed by Yu [13] will be briefly reviewed. In Section 3, the distribution of bending moment in inversion region will be analyzed by the equilibrium of a representative element, and the deformation profile will be studied by finite element analysis. In Section 4, a theoretical model considering curvature change and interaction of bending-tension will be established. And finally, the prediction of current model will be discussed in Section 5.

2. Review of the theoretical model based on deformation theory

In this section, Yu's two-dimensional deformation theory [13] will be briefly reviewed. The axial profile of an external free inversion tube is shown in Fig. 2. Fully clamped at the end near point B, a circular tube with original thickness t_0 and radius R_0 is compressed along axial direction under applied load P; then is inverted from inside to outside. The inversion region AB is assumed to be a semi-circle with radius b. The tube radius is enlarged from R_0 at A to $R_0 + 2b$ at B after inversion. Thereafter, a toroidal small segment dl is changed to dl', after the deformation consists of bending at point A (in axial direction), expansion and bending across region AB (in circumferential direction); and bending at point B (in axial direction). Without considering the deformation history of stress/strain of each material point, by comparing final state with initial state, that is dl' and dl, a simple deformation theory is established based on the energy conservation.

To calculate the *work rate of applied load*, suppose the velocity of small segment dl is v_0, and velocity of dl' is v'. The external work rate done by the applied compression load is,

$$\dot{W} = P(v_0 + v') \tag{1}$$

Then to calculate the *energy dissipation rate* along the circumferential direction $\dot{E}_\varphi$ and along the axial direction $\dot{E}_l$, due to energy conservation,