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First-order generalised beam theory for curved thin-walled members with circular axis

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ABSTRACT

This paper presents a first-order Generalised Beam Theory (GBT) formulation for naturally curved thin-walled members with deformable cross-section, whose undeformed axis is a circular arc with no pre-twist. First, the strain-displacement relations for naturally curved thin-walled members are derived and it is shown how the classic GBT assumptions concerning the strains can be incorporated, namely: (i) Kirchhoff's thin-plate assumption, (ii) Vlasov's null membrane shear strain assumption and (iii) the null membrane transverse extension assumption. The equilibrium equations are obtained in terms of GBT modal matrices and stress resultants. It is demonstrated that, for the so-called "rigid-body" deformation modes (extension, bending and torsion), the GBT equations coincide with those of the Winkler (in-plane case) and Vlasov (out-of-plane case) theories. A standard displacement-based GBT finite element is used to solve a set of representative illustrative examples involving complex local-global deformation. It is shown that the proposed GBT formulation leads to extremely accurate results with a reduced number of DOF and that the GBT modal solution provides an in-depth insight into the structural behaviour of naturally curved members.

1. Introduction

The development of beam theories for naturally curved bars has deserved a significant amount of attention in the past, due to their relevance in many fields of engineering practice — see [1,2], where historical details concerning the early contributions and the classical theories can be found. Developments in this field are still being proposed on a regular basis, even for the geometrically linear case, focusing on finite element technology aspects such as lack of invariance, locking effects and isogeometric approaches [3–7].

This paper presents a first-order (geometrically linear) theory for naturally curved elastic thin-walled bars that extends the classic approaches by introducing arbitrary cross-section in-plane and out-of-plane (warping) deformation. As a first step towards the development of a more general theory for naturally curved and twisted thin-walled bars, it is assumed in this paper that the initial bending curvature is constant (i.e., the beam axis is a circular arc) and that no pre-twist exists. The proposed theory constitutes an important extension of the so-called classic Generalised Beam Theory (GBT) for prismatic bars, which was introduced and initially developed by Schardt and co-workers [8,9] (see www.gbt.info for a list of publications by this group), following the pioneering work of Vlasov [10], and has been widely established as a very efficient, versatile, accurate and insightful mean to assess the structural behaviour of thin-walled bars — see [11–13] and the complete list of publications by the Lisbon-based research group, which can be found at www.civil.ist.utl.pt/gbt.

In the GBT approach, the cross-section kinematic description is based on the superposition of structurally meaningful "cross-section deformation modes", whose amplitudes along the member axis constitute the problem unknowns. According to specific criteria, these deformation modes are subdivided into several subsets (e.g., modes involving rigid-body motions, modes free of membrane shear strains, etc. [14–16]) and are ordered within each subset, so that the first ones are generally the most important to characterize the member structural behaviour. These features make it possible to obtain accurate solutions with only a few deformation modes and even derive semi-analytical or analytical formulae in complex problems. Even in cases where one must resort to fully numerical solutions, using e.g. GBT-based finite elements, the DOF numbers necessary to obtain accurate results are generally much lower than those required by shell finite element analyses. In addition, the modal decomposition of the GBT solution provides in-depth insight into the mechanics of the problems.

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The outline of the paper is as follows. Section 2 presents the fundamental equations of the proposed GBT formulation for naturally curved beams. In particular, Section 2.1 is devoted to obtaining the strain-displacement relations, assuming small-strains and thin walls (Kirchhoff's thin plate assumption is deemed valid). Next, in Section 2.2, the equilibrium equations are derived and written in terms of modal matrices (the classic GBT formalism) and also in terms of stress resultants. Section 2.3 addresses the classic GBT kinematic assumptions, namely null membrane transverse extensions and null membrane shear strains — the so-called Vlasov's assumption, generally acceptable for slender beams with open sections. These two kinematic assumptions are essential for the efficiency of the numerical implementation of the formulation, as they effectively reduce the dimension of the space of admissible deformation modes (i.e., they reduce the number of cross-section DOFs) without significant loss of accuracy and also eliminate shear locking problems (in the case of Vlasov's assumption). Finally, Section 2.4 discusses the particular case of the so-called "rigid-body" modes — axial extension, bending and torsion — and compares the resulting equations with those of the classic theories of (i) Winkler [17], for the in-plane case with coupled axial force and moment, and (ii) Vlasov, for the out-of-plane case with torsion-bending coupling and including warping.

The calculation of the cross-section deformation modes for curved members is discussed in Section 3 and it is shown that the use of Vlasov's assumption introduces a dependence between the in-plane shapes of the modes and the cross-section orientation. Next, in Section 4, details of the finite element implementation of the proposed GBT formulation are provided. A set of representative numerical examples, involving complex local-global deformation phenomena, is presented in Section 5. The paper closes in Section 6, with the concluding remarks.

One final word concerning the notation, which follows closely that introduced in [14], together the vector/matrix forms employed in [18,19]. In this framework, the subscript commas indicate derivatives (e.g., $f_{,x} = \partial f/\partial x$), even if in this paper the prime identifies the derivative with respect to the beam axis arc-length X, i.e. $(\cdot)' = \partial(\cdot)/\partial X$. Finally, it is noted that superscripts $(\cdot)^M$ and $(\cdot)^B$ designate plate-like membrane and bending terms, respectively.

2. First-order GBT formulation for naturally curved members with circular axis

2.1. Strain-displacement relations in wall local axes

Consider the naturally curved thin-walled member shown in Fig. 1 and the associated global cylindrical coordinate system (θ, Z, R), with base vectors $(\boldsymbol{e}_\theta, \boldsymbol{e}_Z, \boldsymbol{e}_R)$. The member axis arc-length coordinate X is also introduced, which lies on the $Z = Z_C$ horizontal plane and has constant curvature equal to $1/R_C$, where C is an arbitrary cross-section "centre" (the intersection of the member axis with each cross-section). In the global cylindrical coordinate system, the displacement field is expressed as $\boldsymbol{U} = u_\theta \boldsymbol{e}_\theta + u_Z \boldsymbol{e}_Z + u_R \boldsymbol{e}_R$ and the small strain-displacement relations are given by (see, e.g., [20])

$$\varepsilon_{\theta\theta} = \frac{u_R + u_{\theta,\theta}}{R}, \tag{1}$$

$$\varepsilon_{RR} = u_{R,R}, \tag{2}$$

$$\varepsilon_{ZZ} = u_{Z,Z}, \tag{3}$$

$$\gamma_{\theta R} = 2\varepsilon_{\theta R} = \frac{u_{R,\theta} - u_\theta}{R} + u_{\theta,R}, \tag{4}$$

$$\gamma_{\theta Z} = 2\varepsilon_{\theta Z} = u_{\theta,Z} + \frac{u_{Z,\theta}}{R}, \tag{5}$$

$$\gamma_{RZ} = 2\varepsilon_{ZR} = u_{R,Z} + u_{Z,R}. \tag{6}$$

Following the usual GBT approach, local axes (x, y, z) are set in each wall, as shown in Fig. 1, where y and z define the wall mid-line and through-thickness directions, respectively, and x is concentric to X. Using the local base vectors $(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z)$, the displacements of the wall are written as

$$\boldsymbol{U}(x, y, z) = u\boldsymbol{e}_x + v\boldsymbol{e}_y + w\boldsymbol{e}_z, \tag{7}$$

where the local displacement components (u, v, w) are related to the cylindrical ones through

$$u_\theta = u, \tag{8}$$

$$u_R = v\sin\varphi + w\cos\varphi, \tag{9}$$

$$u_Z = v\cos\varphi - w\sin\varphi. \tag{10}$$

A coordinate transformation to the local axes is performed, using the rotation angle φ (see Fig. 1) and the relations

$$R = R_0 + y\sin\varphi + z\cos\varphi, \tag{11}$$

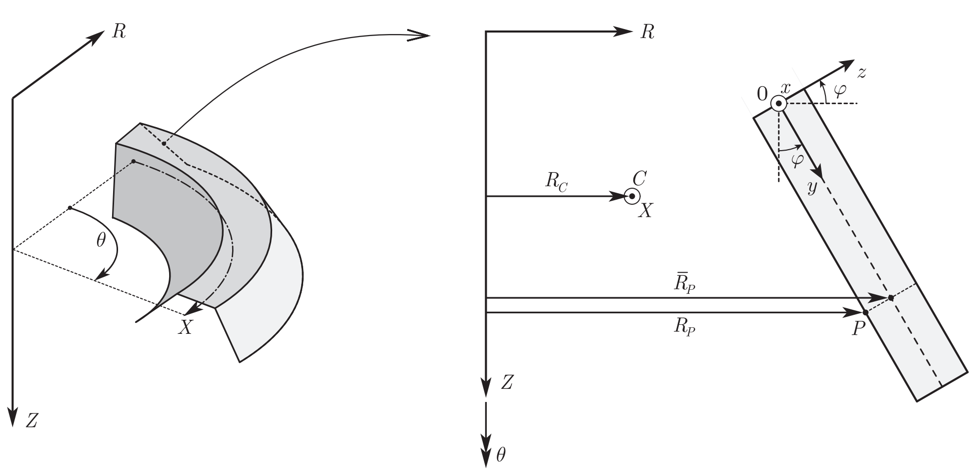

Fig. 1. Global and wall (local) axes for a naturally curved thin-walled beam.