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Shell element for constrained finite element analysis of thin-walled structural members

Sándor Ádány

Budapest University of Technology and Economics, Department of Structural Mechanics, 1111 Budapest, Műegyetem rkp. 3, Hungary

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ABSTRACT

In this paper a novel shell finite element is introduced, specifically proposed for constrained shell finite element analysis. The proposed element is derived from the finite strips used in the semi-analytical finite strip method. The new finite element shares the most fundamental feature of the finite strips, namely: transverse and longitudinal directions are distinguished. Moreover, the new element keeps the transverse interpolation functions of finite strips, however, the longitudinal interpolation functions are changed from trigonometric functions (or function series) to classic polynomials. It is found that the proper selection of the polynomial longitudinal interpolation functions makes it possible to perform modal decomposition similarly as in the constrained finite strip method (cFSM). This requires an unusual combination of otherwise well-known shape functions. If the so-constructed shell finite elements are used to model a thin-walled member, (hence, with using discretization in both the transverse and the longitudinal directions,) modal decomposition can be done essentially identically as in cFSM, whilst the practical applicability of the method is significantly extended (e.g., various restraints, holes, certain cross-section changes can easily be handled). In this paper the focus is on the derivation of the novel shell finite element. Constraining capability is illustrated by some basic examples. Practical application of the novel element will be presented in subsequent papers.

1. Introduction

The finite strip method (FSM) can be regarded as a special version of finite element method (FEM) in which special "finite element"-s are used. The most essential feature of FSM is that there are two pre-defined directions, and the base functions (or: interpolation functions) are different in the two directions. (Typically, though not necessarily and not always, the two characteristic directions are perpendicular to each other). In classical (sometimes also referred as to semi-analytical) FSM, as in [1–4] the structural member to be analyzed is discretized only in one direction (say: transverse direction), while in the other direction (say: longitudinal direction) there is no discretization, i.e., in this direction there is only one element along the member. This is why the dimensions of this "finite element" are typically distinctly different in the two directions, and that is why such an element is called "finite strip" rather than "finite element".

This special finite strip discretization has various consequences. An advantageous consequence is that the total number of elements, therefore the total number of degrees of freedom is much smaller than in case of a classic FEM, which means faster calculation, as well as simpler pre- and post-processing. The price of the calculation efficiency is restricted generality: the analyzed member must be highly regular (e.g., typically it must be straight, prismatic, etc.). A major restriction of the classical semi-analytical FSM is that a certain longitudinal shape function can properly be applied for only a certain problem with given boundary conditions, since in the lack of longitudinal discretization accurate solution can be expected only if the longitudinal interpolation function well represent the real behavior (i.e., if the applied shape function satisfies the differential equation and boundary conditions of the problem).This restriction can (partially) be released if either trigonometric series or splines are used for the longitudinal interpolation. In either case the problem size is significantly increased (compared to FSM), while practical applicability is still limited (compared to FEM).

The original idea of constraining a shell-model is proposed in [5,6] then in [7–10]. The idea is to define special constraints, based on some pre-defined mechanical criteria, the introduction of which enables modal decomposition. Modal decomposition transforms the original displacement field into a set of modal displacements that can solve two basic problems: calculation in a reduced but practically meaningful space (e.g., calculating global buckling directly, by using only a few degrees of freedom), and modal identification (e.g., assigning participation percentages from

E-mail address: sadany@epito.bme.hu

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the modal deformations to a general deformation field).

The constrained finite strip method (cFSM) was first proposed and developed for the semi-analytical FSM with sine-cosine longitudinal shape functions that correspond to locally and globally pinned-pinned end restraints of the thin-walled beam or column. Later other end conditions have also been considered [11,12], but still within the semi-analytical FSM. The method was then generalized to be able to handle general cross-sections [13,14], which also required a more systematic definition of the deformation modes [15]. An attempt to constraining a spline FSM is presented in Ref. [16]. In Refs. [17,18] the constraining technique is applied to shell FEM in the context of a commercial finite element code.

Though all these above-mentioned constrained methods share the same basic mechanical background, there are distinct differences. The advantageous features of cFSM are as follows: (a) the method provides a full modal decomposition (i.e., the whole displacement filed is transformed into a modal system), (b) the mechanical criteria of the modes are exactly satisfied, and (c) adding constraints reduces the DOF number of the problem. However, since cFSM is based on FSM, it has all the restrictions of FSM: for example the member has to be regular, or, FSM is efficient only if the longitudinal shape function is defined specific to the end restraints, which means that arbitrary boundary conditions cannot be handled in an efficient way. The so-far proposed constrained FEM has potentially more general applicability than cFSM, but (a) it provides only partial modal decomposition (that is why modal identification is not readily be handled), (b) the mechanical criteria are satisfied only approximately, and (c) adding constraints increases the DOF number of the problem.

It can be concluded that all of the existing constrained methods possess limitations and/or disadvantages. A better constrained method should provide full decomposition, should satisfy the mechanical criteria exactly, should be generally applicable as much as possible (as far as loading, boundary conditions, etc. are concerned). In this paper the first step toward such a method is presented. A novel shell finite element is introduced. The application of the novel shell element leads to a method which shares (most of) the advantageous features of the original cFSM, while providing significantly extended general practical applicability. The proposed element has a major geometrical limitation of being rectangular and that its local coordinate system must match the longitudinal and transverse direction of the member. Otherwise, no further restrictions are included, that is, the proposed element can be used similarly to any other shell finite elements.

By using the proposed novel shell element various problems can readily be handled: various end and intermediate restraints, nearly arbitrary loading, arbitrary buckling modes including shear buckling or web crippling, certain cross-section changes along the member length, holes. Some of these problems can be solved by other methods, some not. For example, various loading and restraints can be handled by the generalized beam theory (GBT) together with modal decomposition, see e.g. [19–22]. Though attempts to extend GBT for sections with holes are recently reported [23,24], it is still believed that the cFEM based on the here proposed finite element is powerful, since it integrates the advantageous features of all the available modal decomposition methods, and it is based on the shell finite element method which is widely used in research and even in design practice.

In this paper the derivation of the novel shell finite element is presented in detail. Then the constraining capability of the element is illustrated by some basic semi-analytical examples. The examples justify the applicability. The details of the constrained finite element method (cFEM) will be presented in subsequent papers together with various examples to illustrate the advantageous features of the new method.

2. Derivation of the proposed finite element

2.1. Short overview on existing semi-analytical FSM and cFSM

In finite strip method a member is discretized into longitudinal strips, instead of finite element method, which applies discretization in both the longitudinal and transverse directions. In Fig. 1a single strip is shown, along with the typically used local coordinate system and the degrees of freedom (DOF) for the strip, the dimensions of the strip, and the applied end tractions.

By using the simplest longitudinal trigonometric functions, the displacements are approximated as follows.

$$u(x,y) = \left[\left(1 - \frac{x}{b}\right) \ \left(\frac{x}{b}\right) \right] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \sin\frac{m\pi y}{a} \tag{1}$$

$$v(x,y) = \left[\left(1 - \frac{x}{b}\right) \ \left(\frac{x}{b}\right) \right] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \cos\frac{m\pi y}{a} \tag{2}$$

$$w(x,y) = \left[\left(1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3}\right) \ \left(-x + \frac{2x^2}{b} - \frac{x^3}{b^2}\right) \ \left(\frac{3x^2}{b^2} - \frac{2x^3}{b^3}\right) \ \left(\frac{x^2}{b} - \frac{x^3}{b^2}\right) \right] \begin{bmatrix} w_1 \\ \Theta_1 \\ w_2 \\ \Theta_2 \end{bmatrix} \sin\frac{m\pi y}{a} \tag{3}$$

The above formulae represent pinned-pinned boundary conditions. Two important features of the longitudinal shape functions are that the same longitudinal function is used for u and w, and the longitudinal function for v is the derivative of that used for

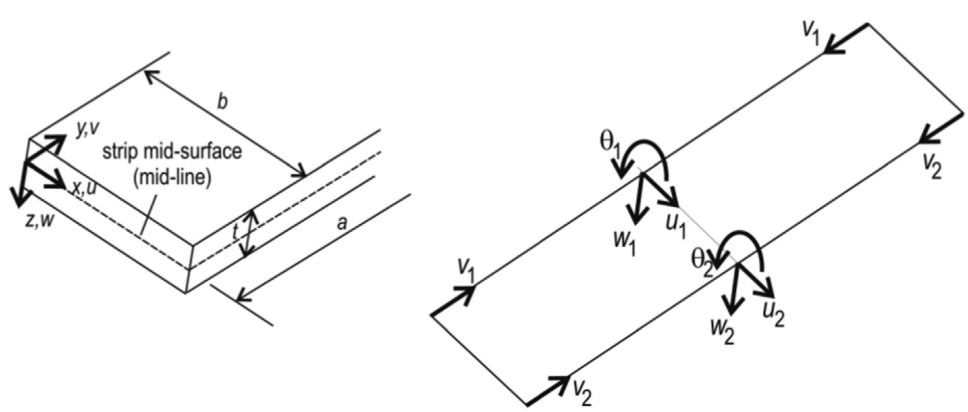

Fig. 1. Coordinates and DOF in finite strip method.