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Stability analysis of thin-walled beams with open section subject to arbitrary loads

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ABSTRACT

In this paper, we present an original work for the examination of the stability of thin-walled beams with open section subject to arbitrary loads. It is based on a 3*D* nonlinear model where the equilibrium and material constitutive equations are established without any assumption on the torsion angle amplitude which leads to strong nonlinearity. In a recently published article [1], we compared this model without simplification to three others models with cubic, quadratic and linear simplifications. The efficiency of the model is confirmed from benchmark solutions. For this reason, we propose in this work to use this model without simplification in the presence of external forces. When these external forces are eccentric they make the nonlinear problem very difficult to solve because the tangent stiffness matrix depends on the load. In presence of arbitrary loads and large torsion context, the right hand side of the equilibrium equations is highly nonlinear and contributes to the tangent stiffness matrix. For this purpose, we use a continuation algorithm based on the Asymptotic Numerical Method ANM, recently published by the authors in [2]. The ANM is a computational tool for solving nonlinear equations numerically; this is achieved by associating the finite element method and a Taylor series expansions technique without any correction and iteration steps. By this way, we compute a large part of the branch by inverting only one stiffness matrix. The efficiency of the present extended model is tested on original applications of open sections thin-walled beams under arbitrary and eccentric loads. A comparison of the obtained results with those computed by Abaqus industrial code is presented.

1. Introduction

Thin-walled beams with open cross sections have a wide range of potential applications in several areas of engineering [3]. They offer high performance with minimal weight and there exists an increasing need for this type of structures. These structures are very sensitive to instabilities and twists. Subjected to external arbitrary eccentric loadings, thin-walled beams with open sections, can exhibit large displacements and large torsion with the presence both of pre-buckling deflexions, warping, bending-bending, torsion-bending and flexural-torsional coupling, shortening and wagners's effects and interaction of buckling and lateral buckling [4–6]. A complete stability nonlinear analysis of these structures must take into account these complex phenomena. The buckling of these beam structures is caused by the coupling among bending, twisting, and stretching deformations of the beam members. Thus the buckling analysis is a subtopic of nonlinear rather than linear models. During these last decades, many finite elements models have been proposed for investigating the stability of thin-walled beams with open constant and variable sections. In these studies, the assumption of finite torsion and cubic trigonometric functions are used. These models have been successfully applied for buckling and lateral buckling analysis of thin-walled beams with open sections. A nonlinear stability analysis model has been recently investigated in [1]. In the model the shortening effect, pre-buckling deformation, large torsion and flexural-torsional coupling are taken into account. The nonlinear and highly coupled equilibrium equations and constitutive law of thin-walled beams with open section are obtained without any hypothesis on torsion angle. The finite element formulation and the use of a 3*D* beam with seven degrees of freedom per node are adopted. The equilibrium paths of these structures in pre and post buckling are get by the Newton-Raphson algorithm. A variant of the Asymptotic Numerical Method

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(ANM) has been proposed recently in [2], for investigating the post-buckling of thin-walled open section beams under various loadings. This ANM variant is based on a 3D nonlinear model of large torsion thin-walled open cross section beams under arbitrary external loads, taking into account the shortening effect, pre-buckling deformation, large torsion and flexural-torsional coupling. In [2], the considered external loads are applied on the middle line.

The aim of this work is to adopt the same alternative in [2] for investigating the buckling and post-buckling behavior of thin-walled open sections beams under external arbitrary eccentric loadings. The effect of eccentric load is studied. The ANM is built by associating the classical finite elements method (FEM) with Taylor series expansions technique. The nonlinear equilibrium equations of thin-walled open section beam have been derived without any approximation on the twist angle. A 3D beam elements having two nodes with seven degrees of freedom are considered in mesh process. The unknowns of the problem are sought in the forms of power series expansions with respect to a path parameter. With this ANM, the nonlinear coupled system is transformed into a sequence of recursive linear systems with the same rigidity matrix which depends also upon the loading. This proposed ANM algorithm is different from that usually considered in previously works based on ANM due to the presence of loading in rigidity matrix. Only one triangulation is needed at each ANM continuation step. The solution branch is obtained without any corrections. The whole solution branch is computed branch by branch via a continuation procedure with an adaptive ANM step length.

The efficiency and accuracy of the proposed ANM are tested on typical examples of open cross section thin-walled beams subjected to external eccentric loads. A comparison of obtained results with those computed by Abaqus industrial code is presented. The obtained results are in good agreement with those, published in the literature [7–10], computed by employing the conventional Newton-Raphson predictor-corrector continuation. This variant of ANM algorithms family is very efficient and less time computationally than the incremental iterative methods, and appears to be robust for solving complicated stability phenomenon of nonlinear thin-walled open sections structures.

2. The nonlinear equilibrium equations

Consider a 3D open section straight thin-walled beam element of length L and cross section $A(x)$ as illustrated in Fig. 1a. The adopted reference system is (G_{xyz}) of center G and of rectangular axes G_x, G_y and G_z such that G_x is the initial longitudinal axis, G_y and G_z are the first and second principal bending axes respectively. The co-ordinates of shear center C located in G_{yz} plane are (y_c, z_c) , those of a point M on the section $A(x)$ are (y, z, ω), with ω is the sectorial co-ordinate which characterizes the warping of the section at point M for non uniform torsion (see Fig. 1b) [3].

In the framework of large displacements, large twist angles and small deformations, the displacements u_M, v_M, w_M of a point M are expressed by the following nonlinear relations [4,11–13]:

$$\begin{cases} u_M = u - y(v' + v'c + w's) - z(w' + w'c - v's) - \omega\theta_x' \\ v_M = v - (z - z_c)s + (y - y_c)c \\ w_M = w + (y - y_c)s + (z - z_c)c \end{cases} \tag{1}$$

where u is the axial displacement of G with G is a center (see Fig. 1a), v and w are the displacements of shear point C in y and z directions, θ_x is the torsion angle. In the model, two trigonometric functions c and s of the twist angle are included as additional variables, they are defined by: $c = \cos(\theta_x) - 1$ and $s = \sin(\theta_x)$. The symbol $(\cdot')$ in equation (1) denotes the derivation with respect to the co-ordinate x. The Vlasov's linear model [3] can be recovered from equation (1) by approximating the trigonometric functions c and s by 0 and θ_x respectively and using linear assumptions. Since the model is concerned with large torsion, the functions c and s are conserved without any approximation in both theoretical and numerical analyses.

We assume in this model that the beam has an elastic behavior and the local and distortional deformations are neglected. The equilibrium equations are obtained from the application of the stationarity of the total potential energy of the structure:

$$\delta U - \delta W_{ext} = 0 \tag{2}$$

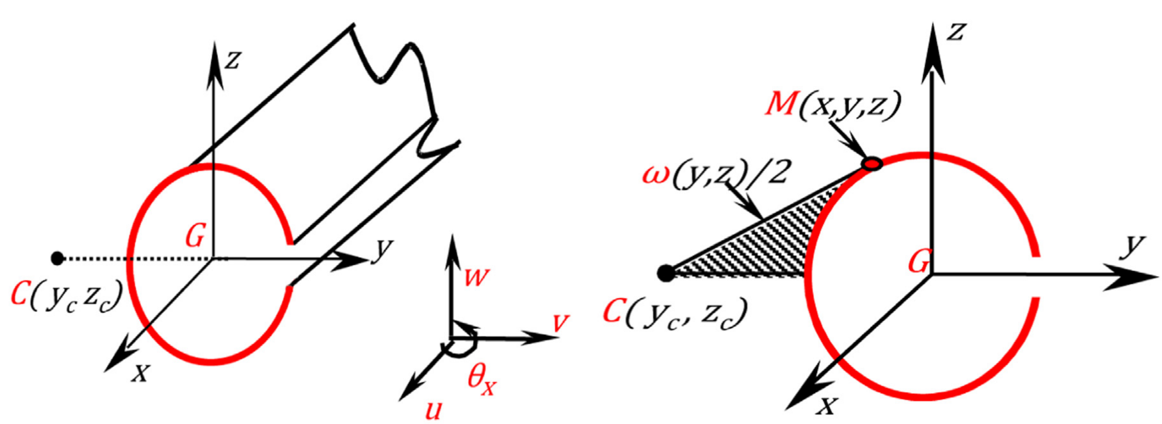
where U is the strain energy and W_{ext} is the work of the applied external forces. The strain energy variation δU is given by the following expression [4,13]:

$$\delta U = \int_L \langle\delta\gamma\rangle\{S\}\, dx \tag{3}$$

with $\langle\cdot\rangle$ denotes the transpose of $\{\cdot\}$. The vector of generalized stresses $\langle S\rangle$ and the vector of generalized strains $\langle\gamma\rangle$ are given by:

$$\begin{cases} \langle S\rangle = \langle N, M_y, M_z, M_{sv}, B_\omega, M_R\rangle \\ \langle\gamma\rangle = \langle\epsilon, -k_y, -k_z, \theta_x', \theta_x'', \tfrac{1}{2}\theta_x'^2\rangle \end{cases} \tag{4}$$

N is the axial force, M_y and M_z are the bending moments about the



(a) Thin-walled open section beam (b) Co-ordinates of the point M on the cross section contour

Fig. 1. Thin-walled open section beam and co-ordinates of the point M on the cross section contour. (a) Thin-walled open section beam. (b) Co-ordinates of the point M on the cross section contour.