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Study on stress relaxation of membrane structures in the prestress state by considering viscoelastic properties of coated fabrics

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ABSTRACT

This paper studies the stress distributions of tensile fabric structures in the prestress state over time to investigate the stress relaxation behavior. To describe the viscoelasticity of coated fabrics, the viscoelastic constitutive law is established based on the generalized Maxwell model. The parameters in the constitutive equation are determined through uniaxial and biaxial stress relaxation tests. The initial state of the actual equilibrium configuration after construction is obtained by means of elastic analysis. By introducing the incremental viscoelastic constitutive equation of coated fabrics into FEM analysis, the stress distributions of equilibrium configuration over time are estimated. A program is developed and the numerical example of stress relaxation analysis of a saddle membrane surface is given. The validity of the suggested analytical method is examined by comparing the numerical results with experimental data of model tests.

1. Introduction

For tensile membrane structures, the initial equilibrium configuration determined by form-finding analysis, is necessary for load analysis. In construction process, the plane membrane strips generated by cutting pattern design are assembled and stretched to the boundary. However, the actual stress and the designed stress of membrane may differ because of the approximation of cutting pattern analysis and the stress relaxation behavior of membrane material. To investigate the stress distribution of the actual equilibrium configuration, the effects of construction process and viscoelasticity of materials need to be considered.

There are two main approaches for simulating the construction process of membrane structures. One analytical method proposed by Tsubota is to establish the equilibrium equation based on the shape of cutting patterns and the linear elastic material model [1]. This method is also used in optimization of cutting pattern generating [2–3]. The other one is the forced displacement method, in which the assembling and stretching order of plane cutting sheets needs to be assumed firstly. Kato used the forced displacement method to simulate the construction process of hyperbolic parabolic membrane structure [4]. However, the analysis process is complicated for some irregular surfaces.

It is vital to establish a proper viscoelastic model in describing the viscoelastic properties of coated fabrics. Several material models have been proposed for coated fabrics, which can be divided into micromechanical models and mathematical models.

Based on the microstructure of coated fabrics, micromechanical models emphasize the local deformation mechanism, such as the crimp interchange, yarn extension and crushing, coating extension and so on. Kato and Pargana proposed the fabric lattice model and an advanced fabric model, which were relatively reliable for describing all the fabric deformation mechanism [5–6]. The formulations of the above models can be directly applied to analysis of tensioned fabric structures. However, the above models focus on the nonlinear elasticity, and the viscous properties have not been involved.

Mathematical models are developed to describe the material response directly based on the experimental data. The plane stress orthotropic model, as the simplest one, is generally used in practical membrane structure design. To determine elastic constants of this model, a biaxial testing method is proposed in Standard of Membrane Structures Association of Japan in 1995 [7]. By using the least-square method, a single set of elastic moduli and Poisson's ratios is obtained to approximate the test data at five different load ratios. Based on the method, Bridgens and Gosling found that the removal of residual strain made the elastic constants more accurate [8]. However, the plane stress orthotropic model is difficult to describe complex non-linear behavior of fabrics with a few of elastic constants. Multi-linear approximation and response surface application are alternative approaches to simulate the nonlinearity [9–10]. The relevant researches require amount of experiment data and large computation times. Galliot

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and Luchsinger proposed on a more simple and reliable model with five biaxial tests. The elastic moduli were formulated as a linear function of the normalized load ratios [11]. Gosling and Bridgens proposed a new philosophy which incorporated test data in structural analysis instead of material models approximation [12]. The above researches emphasize the elasticity without considering viscosity of fabrics. For describing the viscoelastic behavior of membrane materials, the spring-dashpot models are widely used, such as the three-element model, the burger model, generalized viscoelastic models and so on. Moriyama used the burger model to describe the creep behavior of ETFE foil [13]. Fujiwara adopted a three-element model to establish viscoelastic constitutive law by using the biaxial relaxation tests at the stress ratio 1:1 [14]. The above viscoelastic models involve a fairly small number of single elements, which is too limited to describe complex viscoelastic behaviors of materials. Schiessel proposed the fractional models which could be realized physically through hierarchical arrangements of springs and dashpots [15]. Although the fractional model contains a limited set of parameters, the constitutive equation expressed in the fractional integral form leads to complicated mathematic treatments. Compared with micromechanical models, the characteristic parameters of mathematical models can be determined readily by simple experiments.

Based on material models, several investigations of viscoelastic analysis for membrane structures were performed. Argyris added the kevin-voigt model to Meffert's model as viscoelastic model and proposed numerical integration techniques to analyze creep and stress relaxation behaviors of PVC-coated fabric structures [16–17]. The determination of material parameters which is based on uniaxial tests need to be modified by biaxial tests. Kato merged the burger model into the fabric lattice model to analyze the equilibrium configuration in which the construction process and viscoelastic properties were considered [4,18]. However, the large number of required parameters and complicated calculation process of micromechanical model limits its applications. Fujiwara proposed an orthotropic three-parameter model to represent the viscoelastic behaviors of membrane materials [14]. The stress distribution of the structure at the steady state was estimated when the relaxation of membrane terminated, but the stress history over time could not be obtained.

In this paper, a stress analysis method for tensile fabric structures is proposed by means of simulating construction process and involving material viscoelasticity. Both the elastic stress analysis and viscoelastic stress analysis are carried out to find the initial stress distribution of actual equilibrium configuration and the stress relaxation of membrane after construction. In viscoelastic stress analysis, a viscoelastic constitutive formula for membrane materials is developed based on the generalized Maxwell model. The changes of stress distribution of membrane over time are estimated. The validity of the numerical analysis is discussed by stress relaxation experiments of cruciform membrane model.

2. Viscoelastic constitutive law of coated fabrics

2.1. Viscoelastic constitutive law

For coated fabrics, a plane stress anisotropic model is assumed. By considering an arbitrary history of $\{\varepsilon(t)\}$, the stress vector $\{\sigma(t)\}$ can be expressed in the integral form as following:

$$\{\sigma(t)\} = \int_0^t \left[Y\left(t-\tau\right)\right]\frac{\partial\{\varepsilon(\tau)\}}{\partial\tau}d\tau \tag{1}$$

where

$$\{\sigma(t)\} = \left\{\sigma_x(t)\sigma_y(t)\tau_{xy}(t)\right\}^T$$
$$\{\varepsilon(t)\} = \left\{\varepsilon_x(t)\varepsilon_y(t)\gamma_{xy}(t)\right\}^T$$
$$\left[Y(t)\right] = \begin{bmatrix} Y_{11}(t) & Y_{12}(t) & 0 \\ Y_{21}(t) & Y_{22}(t) & 0 \\ 0 & 0 & Y_{33}(t) \end{bmatrix} \tag{2}$$

Here σ_x and σ_y are tensile stresses in fill and warp directions of membrane material and τ_{xy} is shear stress. ε_x, ε_y and γ_{xy} are strains corresponding to the above stresses. $[Y(t)]$ is the relaxation modulus matrix and can be expressed in various forms based on different material models, such as exponential function, power law and so on. The shear stress relaxation is beyond the scope of the paper and ignored here. In this paper, $Y_{33}(t)$ approximately equals the elastic shear modulus and remains constant over time.

Suppose the history of strain $\{\varepsilon(t)\}$ to be a step function with the magnitude $\{\varepsilon_0\}$, beginning at the time zero:

$$\{\varepsilon(t)\} = \{\varepsilon_0\}H(t) \tag{3}$$

with

$$\{\varepsilon_0\} = \left\{\varepsilon_{x0} \quad \varepsilon_{y0} \quad \gamma_{xy0}\right\}^T \tag{4}$$

$H(t)$ is unit Heaviside function. Then Eq. (1) can be simplified as:

$$\{\sigma(t)\} = \left[Y(t)\right]\{\varepsilon_0\} \tag{5}$$

2.2. Identification of material parameters

In this section, the generalized Maxwell model is used and the relaxation modulus matrix $[Y(t)]$ in Eq. (5) is expressed in exponential form. Then the characteristic parameters of the generalized Maxwell model are determined by a series of uniaxial and biaxial stress relaxation tests.

2.2.1. Generalized Maxwell model

For the uniaxial stress relaxation state, when the strain $\varepsilon(t)$ is specified as a step function with the amplitude ε_0, the resulting stress response can be written as

$$\sigma(t) = Y(t)\varepsilon_0 \tag{6}$$

The generalized Maxwell model is assembled by Maxwell elements in parallel, as shown in Fig. 1. A Maxwell element consists of a linear spring and a linear dashpot in series. With the spring and the dashpot following Hooke's law and Newton's law of viscosity respectively, we get the total strain rate $\dot{\varepsilon}$

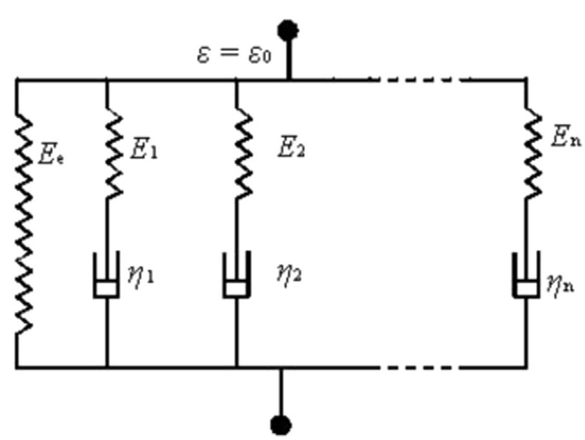

Fig. 1. Generalized Maxwell model.