Thin-Walled Structures 106 (2016) 61–74

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

An incremental numerical method for calculation of residual stresses and strains in cold-formed steel members

H. Amouzegar [a], B.W. Schafer [b], M. Tootkaboni [a,*]

[a] Department of Civil and Environmental Engineering, University of Massachusetts Dartmouth, 285 Old Westport Rd., North Dartmouth, MA 02747–2300, USA
[b] Department of Civil Engineering, Johns Hopkins University, Latrobe Hall 210, Baltimore, MD 21218, USA

ARTICLE INFO

Article history:
Received 19 October 2015
Received in revised form
29 February 2016
Accepted 17 March 2016

Keywords:
Cold-Forming
Residual stress
Residual strain
Cold-formed steel members
Elasto-plastic bending

ABSTRACT

The residual stresses and strains in cold-formed steel members are a result of the manufacturing process. It has been shown that the variation of residual stresses through the thickness of cold-formed steel members is not linear. In this study a numerical algorithm is developed to calculate the through-thickness variation of residual stresses and strains. The algorithm calculates the stresses and strains by viewing the manufacturing process as a combination of elasto-plastic bending and springback in a wide plate under plane strain conditions. In order to calculate the plastic deformations, the Prandtl-Reuss flow rule associated with von Mises yield criterion is used. With regard to satisfying the boundary conditions on the surface, the bisection method is used to find the location of the neutral axis. The results obtained via the proposed algorithm are verified with the available closed formed solutions, finite element analysis results and experimental measurements. A parametric study is performed to evaluate the effect of the coil radius and cross-sectional and material properties on the residual stresses and strains. It is shown that, while in the corner regions the most important parameter is the corner radius, it is the coil radius and yield stress that play a significant role in the variation of residual stresses and strains in the flat regions.

1. Introduction

Residual stresses in hot-rolled steel members, created due to uneven cooling after hot-rolling or welding, do not vary considerably through the thickness of the member [1,2]. In cold-formed steel members, residual stresses are generated as a result of the manufacturing processes i.e. coiling, uncoiling, flattening and cold-forming of the cross section (Fig. 1). In contrast to hot-rolled members, residual stresses due to cold-working vary significantly across the thickness [2,3]. The magnitude as well as cross-thickness variation of residual stresses may play an important role in the mechanical response of structural members that are made by passing steel sheets through a cold forming process. In fact, recent studies (see e.g. [4]) show that the load carrying capacity of cold formed steel columns under bi-axial moments and axial load may change (up to 3%) if residual stresses and strains are included in the nonlinear analysis providing more justification for why design strategies such as Direct Strength Method (DSM) need to include the cold forming effects in their strength predictions (see [5]). In addition, certain features of the mechanical response, such as early yielding on the faces of cold formed steel members or the extent of local buckling in cold formed steel tubes, are directly related to the through thickness variation of residual stresses and strains in these members (see [2,6]).

Efforts have been made to measure residual stresses at the surface of cold-formed steel members [1,7,8]. The through-thickness residual stresses, however, have often been assumed to vary linearly between the measured surface values. Nevertheless, examination of thicker plates [9–11] and theoretical and numerical studies [12,13,6,14–16] have shown that residual stresses in cold-formed steel members have a nonlinear distribution through the thickness. Ingvarsson [12] and Rondal [13] presented a plastic deformation-based incremental numerical algorithm to simulate the bending of a wide plate by using Prandtl-Reuss flow rule and von Mises yield criterion to predict residual stresses at the corners of the cold-formed sections. Kato and Aoki [6] used the same approach to model the plastic deformations created during coiling, flattening and cold-forming sequence to obtain residual stresses and strains in steel tubes. They ignored the normal stresses through the thickness, section thinning and the shifting of the

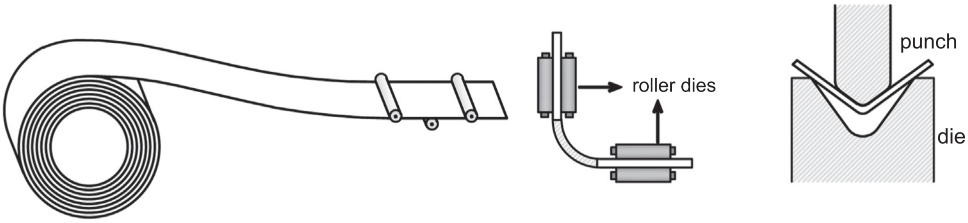
---

* Corresponding author.
E-mail address: mtootkaboni@umassd.edu (M. Tootkaboni).

http://dx.doi.org/10.1016/j.tws.2016.03.019

Nomenclature

t	Thickness
c	Curvatures of inner surface
E	Young's modulus
n_c,n_t	Number of curvature increments and through-thickness segments respectively
R_i	Radius of sheet coil
r, r_i	Radial coordinate and inner radius respectively
ρ	Internal radial coordinate beginning at the inner surface
ρ_o, ρ_{out}	Distance from inner surface to neutral and outer surfaces respectively
ϵ_ϕ, ϵ_r, ϵ_z	Total strains in longitudinal, radial and transverse directions respectively
ϵ_ϕ^e, ϵ_r^e, ϵ_z^e	Elastic strains in longitudinal, radial and transverse directions respectively
ϵ_ϕ^p, ϵ_r^p, ϵ_z^p	Plastic strains in longitudinal, radial and transverse directions respectively
$d\epsilon_\phi$, $d\epsilon_r$, $d\epsilon_z$	Total strain increments
$d\epsilon_\phi^e$, $d\epsilon_r^e$, $d\epsilon_z^e$	Elastic strain increments
$d\epsilon_\phi^p$, $d\epsilon_r^p$, $d\epsilon_z^p$	Plastic strain increments
ϵ_p	Equivalent plastic strain
σ_ϕ, σ_r, σ_z	Stresses in longitudinal, radial and transverse directions respectively
$d\sigma_\phi$, $d\sigma_r$, $d\sigma_z$	Stress increments
ν	Poisson's ratio
σ_y, σ_e	Yield stress from the uniaxial test and equivalent stress respectively
σ_i	Radial pressure at inner surface
H	Strain hardening rate ($d\sigma_e/d\epsilon_p$) in linear hardening



sheet steel coil - uncoiling and flattening roll-forming press-braking

Fig. 1. Manufacturing process for cold-formed steel members.

neutral axis. Quach et al [14] developed an analytical solution to predict the residual stresses and strains due to coiling, uncoiling and flattening. They used Prandtl-Reuss flow rule and von Mises yield criterion to calculate the ratio of transverse stress to longitudinal stress across the thickness and assumed strains, from the coiling to the flattening process, are small enough to ignore strain hardening. A finite element model with plane strain elements was later used to simulate the press-braking procedure [15]. The model has been validated against experimentally measured residual stresses of press-braked thick plates [9,10] and residual strains on the surfaces of lipped channel sections [1]. Moen et al [16] tried to simulate the manufacturing process starting from coiling. They proposed a simplified prediction method to estimate the residual stresses and strains through the thickness of cold-formed steel members. The prediction method was validated against experimental data. It was shown that for the flat parts of the lipped channels, the results are more or less compatible, however, the prediction method overestimated the longitudinal residual stresses at the corners.

This paper attempts to present a comprehensive incremental numerical algorithm to calculate the residual stresses and strains in cold-formed steel members. The algorithm is developed in such a way that it simulates the entire manufacturing process. The main part of the algorithm is solving the elasto-plastic bending problem in polar coordinates under plane strain conditions. It uses the associated Prandtl-Reuss flow rule with von Mises yield criterion to formulate the relationships between plastic strain increments and total strain increments. Strain hardening, section thinning, shift of neutral axis and out-of-plane stresses are all considered. To demonstrate the performance and applicability of the proposed method, the results are compared with analytical predictions of stresses and strains in coiling and flattening, the results of finite element simulations of roll-forming and press-braking and the experimental measurements of residual stresses and strains in press-braked samples. A parametric study is also conducted on the effects of different parameters on the variability in the distribution of residual stresses and strains.

2. Coordinate system

A polar coordinate system as illustrated in Fig. 2 is used where ϕ, ρ and z denote longitudinal, radial and transverse directions respectively. The sign convention for stress and strain is positive for tension and negative for compression.

3. Manufacturing process and the history of stresses

It is assumed that the steel sheet is free of any residual stresses prior to coiling. Plastic deformations during the manufacturing process are thus assumed to be the only cause of residual stresses in cold-form steel members. Consequently, in order to predict residual stresses and strains, it is necessary to follow the manufacturing process step by step. Before being formed into its final shape, the steel sheet from which the cold-formed member is made may experience plastic deformations caused by coiling, uncoiling and flattening processes (simply referred to as the coiling-flattening process). Subsequently, the stresses and strains at the corners of the member's cross-section will change as a result of cold-forming process.

3.1. Coiling-flattening process

After rolling, steel strips are coiled for shipping to the plant. Depending on the radial position in the coil, the sheet might have some residual stresses through the thickness. In preparation for the cold-forming process, the sheet is uncoiled and thus unloaded elastically due to an elastic springback. The residual stresses still present after uncoiling will further change during the flattening