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# Effect of geometrical parameters on mode shape and critical buckling load of dished shells under external pressure

Surya Mani Tripathi [a,b,*], Anup S [b,**], R. Muthukumar [a]

[a] Liquid Propulsion Systems Center (LPSC), ISRO, Valiamala

[b] Indian Institute of Space Science & Technology (IIST), Department of Aerospace Engineering, Valiamala, Thiruvananthapuram 695547, India

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ABSTRACT

A conical shell frustum with flat circular top is called dished shell. These type of shell find application in many aerospace and rocket industries. These type shells undergo a special type of buckling phenomenon similar to arches and spherical caps, which needs to be studied in detail. In the present study, an Eigen value buckling analysis of dished shallow shell is attempted. Two types of dished shallow shells are chosen: with a stiff top, and with a flexible top. Finite element analysis is done to find out the effect of various geometrical parameters such as shell thickness, height, top flat region radius and various boundary conditions on the mode shape and critical buckling load under uniform external pressure. It is found that mode shape and critical buckling pressure changes with variation in geometrical parameters of shells. It is also observed that shells having stiff central flat circular region show different harmonics in conical region for different modes of deformation and the values of critical pressure for different modes are very close to each other. However, for dished shallow shells with flexible top critical buckling pressures are significantly different for various modes except conjugate ones. The numerical analysis results for dished shell with flexible top and fixed boundary condition is compared with analytical solutions of an equivalent spherical cap and for stiff top condition with analytical solutions of an externally pressurized conical frustum. The information obtained from this study, can be used for design improvement and failure mode analysis of dished shells.



## 1. Introduction

Thin shells find application in many engineering areas such as aerospace, marine industries, spacecraft launch vehicles, architecture and beverage industries. The shell is considered to be thin or thick based on the ratio of the shell characteristic dimension to its thickness. For thin shells this ratio is given by $(20 \leq \frac{a}{h} \leq 1000)$. The thin shells of different geometrical shapes such as cylindrical, spherical, conical, toroidal and a combination of these shapes are used for engineering applications. Each of these shells are designed for a specific application and load carrying capabilities. However, in service these structural elements may lose their load carrying ability under compressive load before the maximum material strength is reached. This type of behaviour is undesirable and is referred to as buckling, which is seen as a failure of the system. In some cases, buckling is desirable and is utilised in sensing and load transferring elements.

In literature buckling is classified as Bifurcation and Limit Point Buckling. Bifurcation buckling corresponds to the buckling which occurs before or after the Limit Point of load-deflection curve and latter one corresponds to the Limit Point itself. In general, bifurcation buckling occurring before limit point load is observed in different engineering applications, but bifurcation buckling after limit load is rarely occurring [1]. Limit point buckling is also called as Snap-through buckling. This behaviour of columns, plates, cylinders, cones and spheres have been studied by numerous researchers and listing of these work can be found in review articles [1–4].

In this article we focus on the snap through buckling behaviour, and major studies in this area are discussed briefly. Snap through buckling behaviour is generally observed in case of arches, shallow spherical caps and a special type of conical shells called dished shell. Dished shallow shells and arches/spherical shells have some common behaviour. Therefore, we review some of the important studies in the area of arches/spherical shells. The work by von Karman et al. [5] is the earliest work where snap-buckling finds

* Corresponding author is PhD scholar at: Indian Institute of Space Science & Technology, Valiamala, Thiruvananthapuram 695547, India.

** Principal corresponding author.

*E-mail addresses:* smtripathi.11@iist.ac.in (S.M. Tripathi), anup@iist.ac.in (A. S), r_muthukumar@lpsc.gov.in (R. Muthukumar).

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**Nomenclature**

| | |
|---|---|
| $a$ | Dished shell base circle radius |
| $b$ | Radius of top circular flat region |
| $h$ | Thickness of the conical region of dished shell |
| $h1$ | Thickness of the flat top circular region of dished shell |
| $\alpha$ | Complementary angle of cone semi-vertex angle |
| $H$ | Dished shell height |
| $P$ | Normalised critical buckling pressure/load |
| $E$ | Young's Modulus |
| $r$ | Radial location of any point from center of shell |
| $p$ | Uniform pressure |
| $\mu$ | Poisson's ratio |
| $k$ | Dished shell characteristic parameter |
| $\beta$ | Normalised top flat region radius |
| $\gamma$ | Normalised shell height |
| $\lambda$ | Spherical cap characteristic parameter |
| $\phi$ | Spherical cap included half angle |
| $D$ | Flexural rigidity |
| $R$ | Equivalent radius of spherical cap |
| $P_{cl}$ | Classical buckling pressure of complete spherical shell |
| $P_{cr}$ | Critical buckling pressure for spherical cap |
| $P_{cyl}$ | Critical buckling pressure of cylindrical shell |
| $P_s$ | Critical buckling pressure of conical frustum by Seide P [29] |

mention, where they studied the buckling of spherical cap under external pressure. Later on several analytical studies were conducted by Humphreys [6], Simitses [7] and experimental work by Lock [8], which gave deep insight into the snap buckling of arches. Similarly numerical studies by Willich [9], Thurston [10] and Huang [11] provided mathematical basis for the snap through buckling of spherical shells. Humphreys et al. [12], Budiansky [13] and Lock et al. [14] carried out some interesting experimental studies for spherical shells. Snapping action of spherical cap was studied by Nuri et al. [15] numerically and utilised for design of transducers.

A series of work was carried out in the area of conical shell buckling by Famili [16] and Tani [17] where the problem of buckling under combined axial and pressure loads was tackled. In one of the initial work Wang et al. [18], carried out study for the snap buckling of thin shallow conical shells. In spacecraft launch vehicle, dished shallow shells are used in control components, which isolates the working cryogenic fluid from the driving pneumatic fluid. These are the conical shells with flat top circular region, utilized for actuation of control components (see Fig. 1). Here snap buckling of shell is utilised for actuation of control valves. This behaviour of dished shell is similar to the snapping actions of arches and spherical caps. In this article, we focus on the buckling behaviour of such dished shallow shells. Liu et al. [21,22] has carried out analytical studies for snap buckling phenomenon of dished shells under axis-symmetrically distributed uniform and line load conditions using nonlinear shell theories. They found that snapping action of shell is dependent on shell characteristic parameter, and beyond a particular value only snapping occurs. These analytical studies were further advanced by Zheng et al. [23] for local nonlinear stability under uniformly distributed loads and by Chang-Jiang et al. [24,25] for nonlinear instability modes under circular line load and uniformly distributed load respectively. However, there seems to be no open literature on the effect of geometrical parameters on critical buckling pressure and mode shape under uniform pressure load, different edge boundary

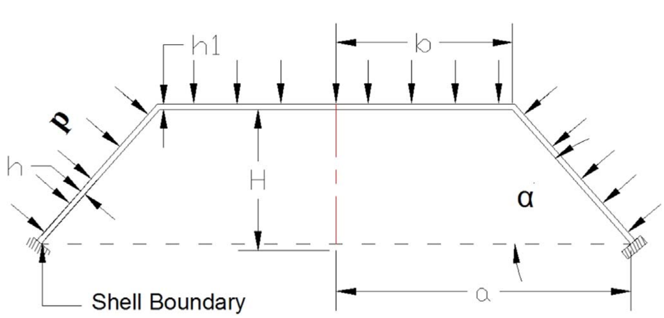


**Fig. 1.** A typical dished shell under external pressure (from [21]).

conditions, nature of flat top circular region and post buckling behaviour of dished type shells, though a limited study on the effect of shell thickness alone was conducted by the authors [27]. Therefore, there is need to study the effect of geometrical parameters like shell thickness, height and top radius on dished shell critical buckling pressure and mode shapes under uniform pressure load. In this article we carry out a parametric study for the effect of geometrical parameters and compare the results for some cases with available literature and experimental results.

In the present study, the numerical model is discussed in Section 2. Details regarding numerical model used for finite element analysis, range of variation in geometrical parameters, parametric expressions for geometrical parameters and critical buckling pressure are given in this section. The analysis results are presented and discussed under Section 3. The results are sub divided into two sub categories based on the dished shell top region flexibility. The results for dished shell with flexible top is discussed in Section 3.1 followed by results for dished shell with stiff top in Section 3.2. The detailed discussion of results for both conditions of dished shell top is given in Sections 3.1.1 and 3.2.1 for shell thickness, in Sections 3.1.2 and 3.2.2 for shell height and in Sections 3.1.3 and 3.2.3 for shell top radius followed by the conclusion in Section 4.

## 2. Numerical model

In the present study, the analysis for critical buckling pressure and mode shape is carried out using finite element software ABAQUS 6.11 [26]. Following assumptions were made to study the present problem.

- The shell of construction is assumed to be of uniform thickness, without imperfection.
- Material of the shell is isotropic, homogeneous and linear elastic.

In the present study dished shells with two different rigidity conditions of top flat circular region are considered. The dished shell with same thickness ($h = h1$) of top and conical shell region is referred to as with "flexible top" and one with relatively rigid top circular region of comparatively higher thickness ($h \ll h1$) than conical portion is referred as with "stiff top".

To find out the critical buckling pressure and mode shapes under uniform pressure load an Eigen value analysis is carried out using Finite Element Method. S4R5 type element in ABAQUS is chosen for the shell. This is a 4-node thin shell element with hourglass control and reduced integration having 5-degrees of freedom per node. Initially a convergence analysis was carried out by varying the number of elements (controlling the characteristic