Thin-Walled Structures 106 (2016) 258–267

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

On the nonlinear stability of eccentrically stiffened functionally graded annular spherical segment shells

Dinh Duc Nguyen*, Huy Bich Dao, Thi Thuy Anh Vu

Vietnam National University, Hanoi, 144 Xuan Thuy, Cau Giay, Hanoi, Vietnam

ARTICLE INFO

Article history:
Received 11 March 2016
Received in revised form
11 May 2016
Accepted 11 May 2016

Keywords:
Nonlinear stability
Eccentrically stiffened FGM annular spherical segment shells
Elastic foundations
External pressure

ABSTRACT

The nonlinear stability of eccentrically stiffened functionally graded (FGM) annular spherical segment resting on elastic foundations under external pressure is studied analytically. The FGM annular spherical segment are reinforced by eccentrically longitudinal and transversal stiffeners made of full metal or ceramic depending on situation of stiffeners at metal-rich or ceramic-rich side of the shell respectively. Based on the classical thin shell theory, the governing equations of FGM annular spherical segments are derived. Approximate solutions are assumed to satisfy the simply supported boundary condition of segments and Galerkin method is applied to study the stability. The effects of material, geometrical properties, elastic foundations, combination of external pressure and stiffener arrangement, number of stiffeners on the nonlinear stability of eccentrically stiffened FGM annular spherical segment are analyzed and discussed. The obtained results are verified with the known results in the literature.

1. Introduction

In recent years, many authors have focused on the static and dynamic of eccentrically stiffened plate and shell structures because these structures usually reinforced by stiffening members to provide the benefit of added load-carrying static and dynamic capability with a relatively small additional weight penalty. In additions, eccentrically stiffened plate and shell is a very important structure in engineering design of aircraft, missile and aerospace industries. As a result, there are many researches on the static and dynamic of eccentrically stiffened shell and plate structures, especially structures made of composite material.

For the eccentrically stiffened plate, the elastic stability of eccentrically stiffened plates [1] was studied by Meiwen and Issam by a finite element model. The formulation was based on the behavior of the plate-stiffener system and accounts for the different neutral surfaces for bending in the x-z and y-z planes. Duc and Cong [2] studied the nonlinear post-buckling of an eccentrically stiffened thin FGM plate resting on elastic foundations in thermal environments by using a simple power-law distribution. An experimental study on stiffened plates subjected to combined action of in-plane load and lateral pressure is described in [3] by Shanmugam et al. The paper [4] presented a periodic concept in stiffened-thin-plates by applying Bloch's theorem. Through the established dynamic equation for periodically stiffened-thin-plate (PSTP), the band gap of PSTP is calculated with the help of center-finite-difference-method (CFDM) by Zhou et al.

Studies on the static and dynamics were carried out with eccentrically stiffened shallow shells made of laminated composite material. For example, Li and Qiao [5] studied the nonlinear free vibration and parametric resonance analysis for a geodesically-stiffened anisotropic laminated thin cylindrical shell of finite length subjected to static or periodic axial forces using the boundary layer theory. In [6], by Sarmila, the finite element method has been applied to analyze free vibration problems of laminated composite stiffened shallow spherical shell panels with cutouts employing the eight-noded curved quadratic iso-parametric element for shell with a three noded beam element for stiffener formulation. For the composite stiffened laminated cylindrical shells, in [7], by Li et al., a layerwise theory was used to model the behavior of the composite laminated cylindrical shells, and the eight-noded solid element is employed to discrete the stiffeners, and then, based on the governing equations of the shells and stiffeners, governing equation of the composite stiffened laminated cylindrical shells was assembled by using the compatibility conditions to ensure the compatibility of displacements at the interface between shells and stiffeners. Li and Yang [8] investigated the post-buckling of shear deformable stiffened an isotropic laminated cylindrical shell under axial compression. Formulation of the dynamic stiffness of a crossply laminated circular cylindrical shell subjected to distributed loads was studied by Casimir et al. [9]. By using the commercial ANSYS finite element software, Less and Abramovich [10] studied the dynamic buckling of a laminated composite stringer stiffened cylindrical panel. Bich

* Corresponding author.
E-mail address: ducnd@vnu.edu.vn (D.D. Nguyen).

http://dx.doi.org/10.1016/j.tws.2016.05.006
0263-8231/

Nomenclature

k	The volume fraction index (non-negative number)
w	The deflection of the annular spherical shell
k_1	The Winkler foundation modulus
k_2	The shear layer foundation stiffness of Pasternak model.
ε_r^0, ε_θ^0	The normal strains
$\gamma_{r\theta}^0$	The shear strain at the middle surface of the spherical shell
$\chi_r, \chi_\theta, \chi_{r\theta}$	The changes of curvatures and twist
s_1, s_2	The distance between eccentrically longitudinal and latitude stiffeners respectively
A_1, A_2	The cross-sectional area of eccentrically longitudinal and latitude stiffeners respectively
d_1, d_2, h_1, h_2	The width and height of eccentrically longitudinal and latitude stiffeners respectively
n_1, n_2	The numbers of eccentrically longitudinal and latitude stiffeners respectively
E_0	The Young's modulus of the stiffeners. $E_0 = E_c$ if the stiffeners are reinforced at the surface of the ceramic-rich, $E_0 = E_m$ if the stiffeners are reinforced at the surface of the metal-rich

et al. [11] presented analytical approach to investigate the nonlinear dynamic of imperfect reinforced laminated composite plates and shallow shells using the classical thin shell theory with the geometrical nonlinearity in von Karman–Donnell sense and the smeared stiffeners technique.

As well as know a functionally graded material (FGM) is a two-component composite characterized by a compositional gradient from one component to the other. In contrast, traditional composites are homogeneous mixtures, and they therefore involve a compromise between the desirable properties of the component materials. Since significant proportions of an FGM contain the pure form of each component, the need for compromise is eliminated. The properties of both components can be fully utilised. This is mainly due to the increasing use of FGM as components of structures in the advanced engineering. For FGM, many researches focused on the static and dynamical analysis of stiffened shallow shells. For example, recently, Duc et al. [12–19] has published several studies on the eccentrically stiffened shell structures made of FGM and the majority of these studies have been synthesized in the book [28]. First example [12] Duc studied the nonlinear thermal dynamic analysis of eccentrically stiffened S-FGM circular cylindrical shells surrounded on elastic foundations using the Reddy's third-order shear deformation shell theory [13], presented nonlinear mechanical, thermal and thermo-mechanical post-buckling of imperfect eccentrically stiffened thin FGM cylindrical panels on elastic foundations [14], investigated nonlinear dynamic response of imperfect eccentrically stiffened doubly curved FGM shallow shells on elastic foundations [15], presented nonlinear post-buckling of imperfect eccentrically stiffened FGM double curved thin shallow shells in thermal environments [16], studied nonlinear response of imperfect eccentrically stiffened ceramic-metal-ceramic S-FGM circular cylindrical shells surrounded on elastic foundations and subjected to axial compression. Bich et al. studied nonlinear post-buckling and dynamic of eccentrically stiffened functionally graded shallow shells and panels [20,21], besides a lot of other researchers by the same authors. In addition, linear static buckling of FGM axially loaded cylindrical shell reinforced by ring and stringer FGM stiffeners has studied by Najafizadeh et al. [22]. Accurate buckling solutions of grid-stiffened functionally graded cylindrical shells under compressive and thermal loads has studied by Sun et al. [23].

The annular spherical shell and annular spherical segment are two of the special shapes of the spherical shells. An annular spherical segment or an open annular spherical shell limited by two meridians and two parallels of a spherical shell. It has become popularly in engineering designs, but despite the evident importance in practical applications, from the open literature that investigations on the thermo-elastic, dynamic and buckling analysis of annular spherical segment is comparatively scarce. In addition, the special geometrical shape of this structure is a big difficulty to find the explicit solution form. Can enumerate some studies of annular spherical shell and segment as Bich and Phuong [24] investigated the buckling analysis of FGM annular spherical shells and segments subjected to compressive load and radial pressure. Most recently, Anh et al. analyzed the nonlinear buckling analysis of thin FGM annular spherical shells on elastic foundations under external pressure and thermal loads in [25], the nonlinear stability of axisymmetric FGM annular spherical shells under thermo-mechanical load in [26,27] investigated the nonlinear stability of thin FGM annular spherical segment resting on elastic foundations in thermal environment.

In this paper, the nonlinear analysis of eccentrically stiffened FGM annular spherical segment shells is investigated. The segment-shells are reinforced by eccentrically longitudinal and transversal stiffeners made of full metal or full ceramic depending on situation of stiffeners at metal-rich side or ceramic-rich side of the shell respectively. The paper analyzed and discussed the effects of material and geometrical properties, elastic foundations and eccentrically stiffeners on the stability of the eccentrically stiffened FGM annular spherical segment.

2. Functionally graded annular spherical shell and elastic foundation

Consider a FGM annular spherical segment or a FGM open annular spherical shell limited by two meridians and two parallels of a spherical shell resting on elastic foundations with radius of curvature R, base radii of lower and upper bases r_1, r_0 respectively, open angle of two meridional planes β and thickness h. The FGM annular spherical segment reinforced by eccentrically longitudinal and transverse stiffeners is subjected to external pressure q uniformly distributed on the outer surface as shown in Fig. 1.

Assume that the FGM segment – shell is made from a mixture of ceramic and metal constituents and the effective material properties vary continuously along the thickness by the power law distribution

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^k, -\frac{h}{2} \le z \le \frac{h}{2},$$
$$V_m(z) = 1 - V_c(z). \quad (1)$$

in which subscripts m and c represent the metal and ceramic constituents, respectively.

According to the mentioned law, the Young modulus can be expressed in the form

$$E(z) = E_m + E_{cm}\left(\frac{2z+h}{2h}\right)^k, -\frac{h}{2} \le z \le \frac{h}{2}. \quad (2)$$

where the Poisson ratio ν is assumed to be constant $\nu(z) = const$