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Free vibration of functionally graded quadrilateral microplates in thermal environment

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ABSTRACT

As a first attempt, the influences of thermal environment together with the geometrical parameters and material properties on the free vibration characteristics of the functionally graded (FG) quadrilateral microplates are investigated. The governing equations are based on the modified strain gradient theory (MSGT) together with the first-order shear deformation theory (FSDT) of plates. Both the temperature dependence of the material properties and the initial thermal stresses are included in the mathematical modeling of the problem. The Chebyshev–Ritz method is employed to extract the free vibration eigenvalue equations from the higher-order governing equations. Chebyshev polynomials in conjunction with suitable boundary functions are used as the admissible functions of the Ritz method to handle the microplates with different set of boundary conditions. After demonstrating the fast rate of convergence and the accuracy of the method, the effects of the temperature rise, length scale parameters, material gradient index, and the length-to-thickness ratio, on the free vibration behaviors of skew and symmetric trapezoidal microplates subjected to different boundary conditions are studied.

1. Introduction

Microplates as fundamental structural elements have been found a wide variety of applications in different branches of modern science and technology, for example in micro/nano electromechanical systems (MEMS/NEMS) [1], biosensors [2,3], biotechnology [4], microbiology [5], radioactivity detection [6], etc. In recent years, the functionally graded materials (FGMs) as advanced nonhomogeneous materials have been used to fabricate microplates with high sensitivity and desired performances for use in MEMS/NEMS [7], atomic force microscopes [8] and sensors [9,10], etc. The physical properties of these materials are functionalized to vary smoothly and continuously in the spatial domain from one material to another material with a specific gradient for the optimum distribution of component materials.

Usually, the structural components made of the FGMs operate in high temperature thermal environments. On the other hand, the induced thermal stresses and also the change in material properties due to temperature rise can reduce the stiffness, and consequently, change the vibrational characteristics of the microstructural elements [11]. Therefore, for a rigorous design and manufacture, it becomes essential to consider these thermal effects when representing a mathematical model for the vibrational motion of the FG microplates in thermal environment.

In order to take into account the size dependence of the micro and nanostructural elements by modifying the classical continuum theory, different higher-order (non-local) continuum mechanics theories such as Eringen's nonlocal elasticity theory [12], couple and modified couple stress theories [13–18], and strain gradient elasticity theory (SGT) [19,20] have been introduced.

In 2003, Lam et al. [21] showed that by introducing a new set of higher-order metrics the number of elastic length scale parameters of the strain gradient theory can be reduced from five to three for the isotropic linear elastic materials. They proved that by using this modified strain gradient theory (MSGT), the equations of motion and the related boundary conditions (including the higher-order boundary conditions) can be derived by means of the variational principle [21]. In spite of the nonlocal elasticity theory of Eringen, a new additional equilibrium equation corresponding to the higher-order stresses is attained in the MSGT. Also, by suitable taken of the length scale parameters, the MSGT can be degenerated to the modified couple stress theory (MCST).

In recent years, the MSGT together with the two-dimensional plate theories such as the classical and shear deformable theories have been employed to study the bending, buckling and free vibration behavior of the microplates; see for example Refs. [22–30]. However, to the best of authors' knowledge, the most of these interesting research works concerned with the mechanical behavior of the microplates and only in the limited works [29–31], the

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thermal effects on the global behavior of the microplates were studied. In addition, in these works the influence of thermal environment on the rectangular and annular/circular microplates, and based on the MCST were investigated [29–31]. But, in some cases, to improve the physical performance or due to the design restriction, the designer may be forced to adopt the non-rectangular and non-circular/annular microplates such as the skew and trapezoidal plates.

In the present study, as a first endeavor, the influences of thermal environment on the free vibration characteristics of the FG quadrilateral microplates subjected to different boundary conditions are studied. In order to accurately model the size effect and the global vibrational behavior of the microplates, the MSGT in conjunction with the FSDT of plates are used to formulate the problem. Both the initial thermal stresses and the temperature dependence of the material properties are included in the mathematical modeling of the problem. It is assumed that the material properties vary in the thickness direction. Since it is difficult to solve the higher-order governing equations of the MSGT analytically, especially for quadrilateral FG microplates, a high accurate and fast convergent approximate method becomes necessary to find the natural frequencies. On the other hand, the computational efficiency and accuracy of the Chebyshev–Ritz method for solving complicated structural problems has been established [32,33]. Therefore, this method is adopted to extract the free vibration eigenvalue equations of quadrilateral FG microplates subjected to different boundary conditions in thermal environment. The fast rate of convergence of the method is demonstrated, and by comparing the obtained results with those available in the open literature in the limit cases, its accuracy is also demonstrated. Then, the effects of the temperature dependence of the material properties, initial thermal stresses, temperature rise, material length scale parameters, material property gradient index, and the thickness-to-length ratio on the vibrational behavior of the FG skew and symmetric trapezoidal microplates with different set of boundary conditions are studied. Also, in some cases, the results of the MCST and the MSGT are compared and discussed.

2. Mathematical modeling

The geometry of the FG quadrilateral microplates under consideration is shown in Fig. 1. The Cartesian coordinate system with coordinate variables (x, y, z), which is placed on the mid-plane of the microplate, is used to label the material points of the microplates. It is assumed that the microplates are composed of two different phases (i.e., metal and ceramic phases) and the volume fractions of their compositions vary continuously in the thickness direction. In the following, the basic governing equations and the solution procedure are described.

2.1. Effective material properties and temperature distribution

The rule of mixture is a simple and well established model to estimate the effective material properties of the FGMs [34]. According to this micromechanical model, a typical effective material properties P can be written as

$$P = P_m + \left(P_c - P_m\right)V_c \tag{1}$$

where P_m and P_c are the corresponding material properties of the metal and ceramic components, respectively. Without loss of generality of the formulation, in this study it is assumed that the material distribution follows a power law in the thickness direction,

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n, \; V_m = 1 - V_c \tag{2a,b}$$

where V_m and V_c are the volume fraction of the metal and ceramic phases, respectively; n is the material property graded index or power law index, which is a positive real number. Hereafter, the subscripts c and m are used to denote the material parameters of the ceramic and metal phases, respectively.

It is assumed that a typical temperature-dependent material property 'Q' of the FG microplate constituents, vary with temperature as [35].

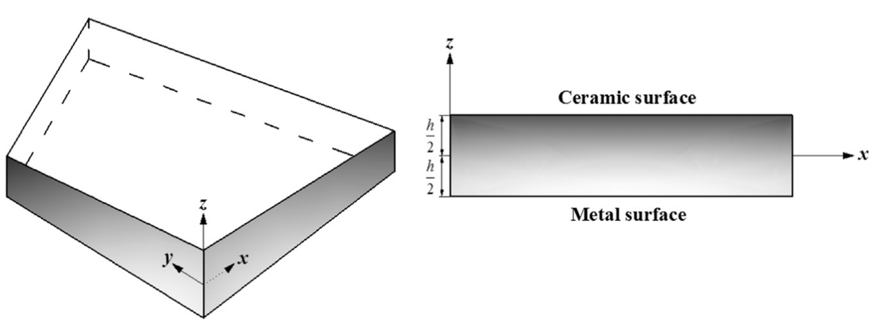

Fig. 1. An arbitrary straight-sided quadrilateral microplate.

Table 1.
The temperature-dependent coefficients of material properties for ceramic (Si_3N_4) and metals (SUS304) [33].

	Material	Q_{-1}	Q_0	Q_1	Q_2	Q_3
E	SUS304	0	201.04 (GPa)	3.079×10^{-4}	-6.534×10^{-7}	0
	Si_3N_4	0	348.43 (GPa)	-3.07×10^{-4}	2.16×10^{-7}	-8.946×10^{-11}
ν	SUS304	0	0.3262	-2.002×10^{-4}	3.797×10^{-7}	0
	Si_3N_4	0	0.24	0	0	0
ρ	SUS304	0	8166 (kg/m^3)	0	0	0
	Si_3N_4	0	2370 (kg/m^3)	0	0	0
α	SUS304	0	12.33×10^{-6}(1/K)	8.086×10^{-4}	0	0
	Si_3N_4	0	5.87×10^{-6}(1/K)	9.095×10^{-3}	0	0
k	SUS304	0	12.04 (W/mK)	0	0	0
	Si_3N_4	0	9.19 (W/mK)	0	0	0