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Simplified semi-analytical model for elastic distortional buckling prediction of cold-formed steel flexural members

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ABSTRACT

The expressions for elastic distortional buckling stress predictions available in literature are used with reasonable accuracy, but are either iterative or involve increased calculation effort for design. It is an accepted fact among researchers that these expressions are not evaluated enough for the distortional stress prediction of sections with complex lip stiffeners. In this paper, a candidate model for distortional buckling stress predictions is presented which is semi analytical in nature and is simple to incorporate in the direct strength method (DSM) of cold-formed steel design. The proposed expression incorporates the effect of complex lip stiffeners on the elastic distortional buckling capacities of cold formed steel flexural members. In the proposed model, the (i) translational stiffness at lip-flange junction and (ii) rotational stiffness at the flange-web junction, are derived from regression analysis of wide range of cross sectional dimensions. The proposed method is calibrated with semi-analytical finite strip method presented in the literature and this formulation has been demonstrated to be good in comparison with recently published numerical based distortional buckling predictions.

1. Introduction

The migration in design philosophy of cold-formed steel sections from traditional effective width method to direct strength method necessitates the evaluation of elastic buckling stresses such as (i) local buckling, (ii) distortional buckling and (iii) overall Euler buckling stress using numerical methods like finite element method, finite strip method or by using analytical expressions. Analytical expressions are available in Timoshenko and Gere [1] for the evaluation of elastic local and Euler (global) buckling stresses whereas distortional buckling predictions are usually based on analytical model of flange-lip combination with rotational and translational stiffness at junction point. Distortional buckling phenomenon is characterized by the rotation of the flange-lip combination about the flange-web junction and the analytical model is designed to incorporate this mechanism.

There are several papers on the prediction of distortional buckling stresses which is applicable for the design procedures presented in codes of practice. One of the earliest work related to distortional buckling was on steel storage rack columns by Hancock [2], providing design chart for buckling coefficient and half buckling wavelength for a range of section geometries. Kwon and Hancock [3] provided design equations for distortional buckling and post buckling strength of lipped channel compression members based on experimental investigation. Later Hancock et al. [4] proposed strength design curves for thin walled sections under distortional buckling by incorporating the influence of local buckling on distortional buckling. Modifications to EC3 method for predicting the distortional buckling stress using generalized beam theory (GBT) are available in Kesti and Davies [5] and provides stress values comparable with experimental and theoretical results. He and Zhou [6] proposed strength design curve for distortional buckling of cold-formed steel columns under axial compression. An effective width formula for distortional buckling was also proposed in [6], which shows better agreement with current direct strength method (DSM) predictions than the existing effective width method.

As far as the efforts to use numerical methods are concerned, Lau and Hancock [7] analysed the inelastic distortional buckling strength of thin walled columns using spline finite strip method. A nonlinear finite element model calibrated using experiments has been simulated by Yu and Schafer [8] for application in direct strength method. Jiang and Davies [9] studied the behaviour of restrained purlins under distortional buckling mode using GBT. Experimental work on distortional buckling of beams include the tests by Yu and Schafer [10] on channel and zed section beams under bending where by the sections are constrained to buckle in distortional buckling mode by effectively arresting local and lateral torsional buckling in the testing procedure.

For works related to analytical methods on elastic distortional

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buckling prediction, Lau and Hancock [11] derived analytical expression for elastic distortional buckling stress of compression members using flexural torsional buckling theory developed by Timoshenko and Gere [1] and Vlasov [12]. The derivations were based on approximate model of flange-lip combination rotating about flange-web junction. The theory has been extended by Hancock [13] to flexural members wherein a design method for flexural members was provided. Iterative calculations are involved in the determination of rotational stiffness terms provided in [11,13]. Analytical expression for sections with laterally unsupported compression flanges for evaluating elastic distortional buckling stress based on the assumption of beam on elastic foundation was presented by Serrette and Pekoz [14]. A closed form solution for distortional buckling based on a model of flange-lip system, with tension end of web treated as pinned was presented by Davies and Jiang [15]. Rogers and Schuster [16] compared various analytical models on distortional buckling of flexural members and established an improved prediction by Hancock model on elastic distortional buckling strength. The analytical model proposed by Schafer and Pekoz [17] considered the interaction model of web and flange by incorporating the elastic stiffness and geometric stiffness portion of both flange and web for distortional buckling strength calculation. The web stiffness contribution for the formulation were derived using numerically based semi-analytical finite strip method. Teng et al. [18] provided analytical expression for evaluating elastic distortional buckling strength of sections subjected to axial compression and biaxial bending by extending Lau and Hancock [11] method. Silvestre and Camotim [19] provided analytical formula for distortional critical length and buckling stress for cold-formed steel channel and zed section members under uniform compression, pure bending and both using generalized beam theory. A model for calculating distortional buckling stress of channel, zed and sigma section under compression or bending about an axis perpendicular to the web was proposed by Li and Chen [20]. The analytical model consists of lip-flange combination incorporating a translational spring instead of rotational spring as adopted in Lau and Hancock [11] model. Tong et al. [21] proposed a unified analytical model for the distortional and lateral buckling analysis of channel beams. Salient points of development in the analytical models for distortional buckling stress prediction are briefly compared in the next section.

2. Critical evaluation of existing formulations on elastic distortional buckling stress

The mechanics involved in the distortional buckling of cold-formed steel flexural member is shown in Fig. 1. In an analytical model, the deformation of cross section as a result of distortional buckling is idealized as flange-lip combination under uniform compression with rotational (k_ϕ) and translational (k_x) stiffness. The equations of equilibrium by considering forces along x and y direction and moment about shear centre are formulated using flexural torsional buckling theory. The partial differential equations are solved to determine the elastic distortional buckling load. The role of k_ϕ is to incorporate the rotational restraint offered by web on web-flange junction and the resistance offered by web is also dependent on the fixity at tension flange and stress gradient in the web. The effect of k_x has been neglected in majority of reported research work [11,13,15,20] and its effect on elastic distortional buckling strength of flexural members has been neglected in this paper also. The existing formulations are based on flanges under uniform compression and hence the capability of these formulations to capture the effect of stress gradient in the flange on distortional buckling is assessed here in this study.

A comparative study of existing analytical models on elastic distortional buckling prediction of flexural members has been performed. Two analytical models for distortional buckling of flexural members; Hancock [13] and Davies and Jiang [15] (which will hitherto be called as Hancock model and Davies model, in the present study) are compared for the evaluation of elastic distortional buckling stress under flexure. The geometry of lipped channel cross sections for the comparison of analytical models are taken from the work of Hancock [13] and Schafer and Pekoz [17] and the results are compared with finite strip analysis (FSM) results using the software CUFSM developed by John Hopkins university [22]. The comparison of distortional buckling stress predictions of analytical models with finite strip results with variation in flange width to lip depth ratio (b/c) in a non-dimensional form and the statistical comparison of results is shown in Fig. 2. It should be noted that the range of sections that are considered for the comparison has compactness ratio (b/h) varying from 0.13 to 0.75. The flange width to lip depth ratio (b/c) considered is between 2 and 16. In direct strength method, a value of $b/c > 1.4$ is accepted as within the pre-qualified range.

Elastic distortional buckling calculation using analytical models are comparable with finite strip results over a range of cross section dimensions and relatively better estimate of results are available for b/c value greater than 5. For b/c value less than 5, the analytical predictions were mostly lower bound to finite strip results and the percentage variation is nearly 40%. For a few section geometries, Hancock and Davies models produces negative rotational stiffness values at web-flange junction resulting in very high elastic distortional buckling strength prediction. In the expression presented by Hancock model and Davies Model in literature, one could see that the multiplier bracketed term for the rotational stiffness k_ϕ, represents the reduction in the flexural stiffness offered by the web due to axial stresses. However when this term becomes negative, the reduction is completely neglected. This in turn results in prediction of higher capacities, which is evident from Fig. 2.

Comparative study has been done on lipped channel cross section to assess the effect of section geometries on distortional buckling stress with compactness ratio (b/h), by varying the width (b) of the section keeping other cross sectional dimensions constant ($h = 300$ mm, $c = 50$ mm and $t = 1$ mm). The values calculated using theoretical models are compared with distortional buckling stress calculated using finite strip analysis as shown in

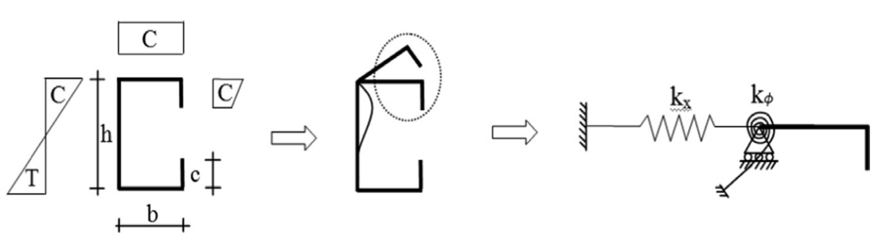

Fig. 1. Distortional mechanics of cold-formed steel beams under flexure [11]].