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# Strength and stability of geometrically nonlinear orthotropic shell structures

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ABSTRACT

The article presents a methodology for the study of shell structure strength and stability. The basis of the study is a geometrically nonlinear mathematical model, which takes into account the transverse shifts and orthotropy of material. The model is presented in dimensionless parameters in the form of the total energy potential functional and can be used for different types of shells of revolution.

The model is studied by using an algorithm based on the Ritz method and the method of solution continuation according to the best parameter (MSCBP), which allows for obtaining the values of the upper and lower critical loads and examining the supercritical behavior of designs.

In accordance with an algorithm, the computer program has been developed and a comprehensive study of the strength and stability of shallow shells (which are square in plane), cylindrical, and conical panels has been explored. The load loss of strength and buckling load values have been obtained, and their relationship to one another has been demonstrated.



## 1. Introduction

In modern building, as well as in shipbuilding, mechanical engineering, aviation, and other industries, structures in the form of shells have wide application [1–12]. Currently there are composite materials [13–19] (CFRP, GRP etc.), which have high strength, fire resistance, chemical and corrosion resistance, lightness, and their application in the design of shell structures deserves much attention. Given that the reinforcement elements in the material are often placed along curved axes of the shell's coordinate system, such structures are to be considered orthotropic [20,21].

Renewed interest in the study of shell structures in recent years has arisen not only due to the emergence of promising new materials, but also, above all, due to the development of computer technologies, which allow one to take a fresh look at the nonlinear problems of shells [22,23].

Composite materials are widely used in machine building, shipbuilding and rocketbuilding, but in building, used as coatings of span structures, such materials are still not widely used because of their high cost and the lack of research on such structures.

In considering the problems of shell stability it is important to analyze the strength of the material, given that after the flowing deformation or brittle fracture stress, irreversible changes occur, and study of the stability of the structure in the linear elastic formulation is invalidated.

Therefore the combined research of strength, stability and supercritical orthotropic materials shell behavior is topical, and it is based on the most accurate mathematical models of their deformation, effective computational algorithms, and specially developed software.

Shell resistance has been studied by many authors, but almost all publications relate to the study of isotropic shells. The works of Rikards and Teters [20] are among the first studies on the stability of orthotropic shells. In these works a mathematical model based on the hypothesis of Kirchhoff – Love was used; however, as shown by experimental research, it is necessary to consider lateral shifts in the study of the stability of such shells. The works by Reddy, Guz and Babic, Maksimyuk and Chernyshenko and others have generated great theoretical interest.

The present state of the different divisions of shell theory is well represented in review articles and monographs by Carrera [1], Qu [24], Reddy [25], Ventsel and Krauthammer [10], Golushko and Nemirovsky [4], Karpov [26,27], Grigolyuk and Kulikov [28], Maksimyuk and Chernyshenko [21] et al. [29–32]. A number of publications is devoted to the review on works on shell stability [33–35].

Composite material shell stability is examined in the works of Trushin [29]. Experimental data of the determination of the limit values of composite material shell stresses are presented in the works of Smerdov et al. [13].

Special attention should be paid to conical shells, which for the most part are widely used in aviation technology and engineering,

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but also used in construction [12]. In the field of the shell stability research, the works of Mushtari, Alumyae, Grigolyuk, Sachenkov et al., as well as works [36–39] should be noted. The problem of conical shell stability was reduced to finding the eigenvalues of the system of partial differential equations with variable coefficients, and it was shown that the solution must be sought approximately [40].

One of the previously used approaches to solve this problem was the reduction of the conical shell to the cylindrical shell. The radius of the cylindrical shell is adopted as a cross between a large and small radius of the conical shell. This technique proved successful in the calculation of shells with a small conical angle [transformed], but with its increasing conical shell structure specificity its stability begins to be affected, and this approach is no longer acceptable.

Compared to calculating cylindrical shells, it is more difficult to explore such constructions. This is manifest primarily in the complication of the geometrical relationships which link displacements and deformations. If we substitute in them the formulas of curvature and the Lame parameters ( $A = 1,\ B = x\cdot \sin\theta$, $k_x = 0,\ k_y = \frac{ctg\theta}{x}$), due to the formula's dependence on the variable $x$, the complexity of further calculations increases significantly.

Recent works in this area should include an article by Shadmehri et al. [38], where closed conical shells made of composite materials are considered; however the mathematical model is based on first order theory, and geometric nonlinearity is not taken into account.

In most works concerning shell stability, cylindrical shells are the most studied, but supercritical behavior and the relationship between stability and strength are not investigated.

The purpose of this work is to study the strength and stability of geometrically nonlinear orthotropic shell structures based on the proposed methodology.

## 2. Methodology used and related equations

### 2.1. Governing equations

A mathematical model of deformation of shells consists of three groups of relations:

- geometric relations linking deformations and displacements;
- physical relations between stresses and strains;
- the total energy functional of deformation of the shell, from the minimum of which the equations of equilibrium are derived.

Fig. 1 shows the general view of the thin-walled shell with axes of the local coordinate system $x, y, z$ (orthogonal coordinate system in the middle surface of the shell structure; $x, y$ – the curvilinear coordinates directed along the lines of the principal curvatures, $z$ – coordinate directed towards the concavity of the shell surface perpendicular to the middle).

For shells of revolution of the general form we can introduce the following dimensionless parameters [41]:

$$\xi = \frac{x}{a},\quad \bar{a}_1 = \frac{a_1}{a},\quad \eta = \frac{y}{b},\quad \lambda = \frac{aA}{bB},\quad k_\xi = hk_x,\quad k_\eta = hk_y,$$
$$\bar{U} = \frac{aUA}{h^2},\quad \bar{V} = \frac{bVB}{h^2},\quad \bar{W} = \frac{W}{h},\quad \bar{\Psi}_x = \frac{\Psi_x aA}{h},\quad \bar{\Psi}_y = \frac{\Psi_y bB}{h},$$
$$\bar{\sigma}_x = \frac{\sigma_x(1-\mu_{12}\mu_{21})a^2A^2}{E_1h^2},\quad \bar{\sigma}_y = \frac{\sigma_y(1-\mu_{12}\mu_{21})a^2A^2}{E_2h^2},\quad \bar{\tau}_{xy} = \frac{\tau_{xy}a^2A^2}{G_{12}h^2},$$
$$\bar{P} = \frac{a^4A^4q}{h^4E_1},\quad \bar{A} = \frac{aA}{h},\quad \bar{B} = \frac{bB}{h},\quad \bar{z} = \frac{z}{h},\tag{1}$$

where $a, b$ – linear dimensions of the shell in the directions of axes $x$ and $y$ respectively [m], $\lambda$ – dimensionless coefficient; $\xi, \eta, \bar{z}$ – new (dimensionless) coordinate system of the shell; $U = U(x, y), V = V(x, y), W = W(x, y)$ – displacements of points of the middle surface of the shell along axes $x, y, z$ [m]; $\Psi_x = \Psi_x(x, y),\ \Psi_y = \Psi_y(x, y)$ – rotation angles of normal in the planes $xOz$ and $yOz$ respectively; $k_x = \frac{1}{R_1},\ k_y = \frac{1}{R_2}$ – the major curvatures of shell along axes $x$ and $y$ [1/m]; $R_1, R_2$ – the major radii of curvature of shell along axes $x$ and $y$ [m]; $A, B$ – Lame parameters, which characterize the geometry of the shell; $h$ – shell thickness [m]; $q = q(x, y)$ – outward transverse load [MPa].

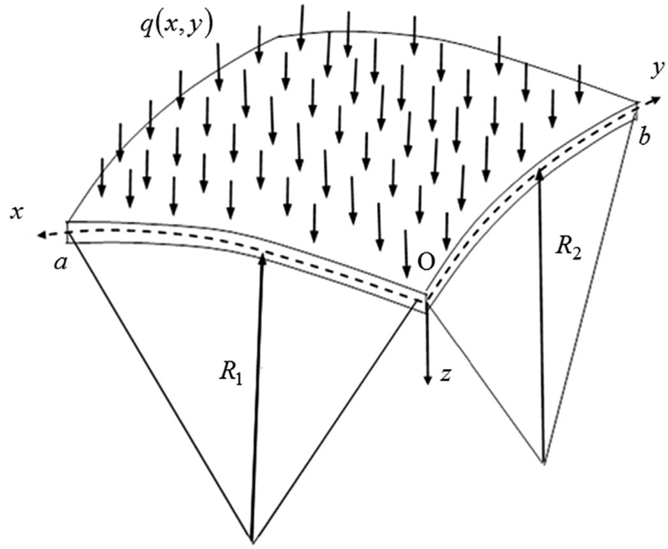


**Fig. 1.** General view of the thin-walled shell with axes of the local coordinate system.

Geometric relations in the middle surface of the shell, taking into account the geometric nonlinearity [42], using the dimensionless parameters (1), take the following form

$$\bar{\varepsilon}_x = \frac{\partial \bar{U}}{\partial \xi} - k_\xi \bar{A}^2\bar{W} + \frac{1}{2}\bar{\theta}_1^2;\quad \bar{\varepsilon}_y = \lambda^2\frac{\partial \bar{V}}{\partial \eta} + \frac{1}{\bar{B}}\frac{\partial \bar{B}}{\partial \xi}\bar{U} - k_\eta \bar{A}^2\bar{W} + \frac{1}{2}\lambda^2\bar{\theta}_2^2;$$

$$\bar{\gamma}_{xy} = \frac{\partial \bar{V}}{\partial \xi} + \frac{\partial \bar{U}}{\partial \eta} - \frac{1}{\bar{B}}\frac{\partial \bar{B}}{\partial \xi}\bar{V} + \bar{\theta}_1\bar{\theta}_2;\quad \bar{\gamma}_{xz} = k\ f(\bar{z})\left[\bar{\Psi}_x - \bar{\theta}_1\right];$$
$$\bar{\gamma}_{yz} = \lambda k\ f(\bar{z})\left[\bar{\Psi}_y - \bar{\theta}_2\right].$$

Here $f(\bar{z})$ – function which characterizes the distribution of stresses $\tau_{xz}$ and $\tau_{yz}$ over the thickness of shell $f(\bar{z}) = 6\left(\frac{1}{4} - \bar{z}^2\right);\ k = \frac{5}{6};$

$$\bar{\theta}_1 = -\left(\frac{\partial \bar{W}}{\partial \xi} + k_\xi \bar{U}\right);\quad \bar{\theta}_2 = -\left(\frac{\partial \bar{W}}{\partial \eta} + k_\eta \bar{V}\right).$$

Functions of change of curvature and torsion

$$\bar{\chi}_1 = \frac{\partial \bar{\Psi}_x}{\partial \xi};\quad \bar{\chi}_2 = \lambda^2\frac{\partial \bar{\Psi}_y}{\partial \eta} + \frac{1}{\bar{B}}\frac{\partial \bar{B}}{\partial \xi}\bar{\Psi}_x;\quad 2\,\bar{\chi}_{12} = \frac{\partial \bar{\Psi}_y}{\partial \xi} + \frac{\partial \bar{\Psi}_x}{\partial \eta} - \frac{1}{\bar{B}}\frac{\partial \bar{B}}{\partial \xi}\bar{\Psi}_y\ .$$

At the linearly elastic deformation, physical relations for orthotropic shells of revolution [43] with the dimensionless parameters (1) can be written as

$$\bar{\sigma}_x = \bar{\varepsilon}_x + \mu_{21}\bar{\varepsilon}_y + \bar{z}\left(\bar{\chi}_1 + \mu_{21}\bar{\chi}_2\right);\quad \bar{\sigma}_y = \bar{\varepsilon}_y + \mu_{12}\bar{\varepsilon}_x + \bar{z}\left(\bar{\chi}_2 + \mu_{12}\bar{\chi}_1\right);$$

$$\bar{\tau}_{xy} = \lambda\left[\bar{\gamma}_{xy} - 2\bar{z}\bar{\chi}_{12}\right];\quad \bar{\tau}_{xz} = Gkf(\bar{z})\bar{A}\left[\bar{\Psi}_x - \bar{\theta}_1\right];\quad \bar{\tau}_{yz} = kf(\bar{z})\bar{A}\lambda\left[\bar{\Psi}_y - \bar{\theta}_2\right].$$

Functionality of total potential energy of deformation [44] is written as follows: