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Vibrations and stability of a loaded side-cracked rectangular plate via the MLS-Ritz method

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ABSTRACT

This study deals with vibration and stability analyses of a rectangular plate with a side crack based on classical thin plate theory using the famous Ritz method with admissible functions that are constructed by the moving least square (MLS) method with enriched basis functions. The enriched basis functions consist of regular polynomial functions along with crack functions that appropriately describe the stress singularities at the crack tip and show the discontinuities of displacement or slope crossing the crack. Comprehensive convergence studies on the stress intensity factor, buckling loads and vibration frequencies of a cracked rectangular plate under uniform loading at its two opposite edges are carried out and demonstrate the accuracy and efficiency of the presented approach by comparing the present results with previously published ones. Finally, the present approach is applied to investigate the effects of location, length and orientation of side cracks on the buckling loads, vibration frequencies and mode shapes of cracked rectangular plates. The in-plane boundary conditions are normal traction prescribed along two opposite edges and free along the other edges. Two out-of-plane boundary condition combinations are considered. One is simply supported along all the edges, and the other is simply supported along the two loaded edges and free along the other edges.

1. Introduction

The flat plate is a very common component in engineering structures. Rectangular plates subjected to external in-plane loads have been often found in civil, mechanical and aerospace structures. Sharp corners of openings, cutouts, welding and irregular loads very likely cause initiation and propagation of cracks. A crack in a plate can make its dynamic characteristics significantly different from those for an intact plate. The redistribution of in-plane internal stress resultants due to the existence of a crack can further change the behaviors of a plate. Consequently, it is interesting and important to investigate the dynamic characteristics and stability of cracked plates subjected to in-plane loads. Linear analyses for such plates involve two main steps. The stress resultants in a plate under in-plane loading are first determined. Then, the vibration frequencies of the plate with those stress resultants as initial stresses are found. The loads resulting in zero nature frequencies are the buckling loads. Notably, the effects of crack closure due to preloads are not considered in the linear analyses.

A vast pool of literature has investigated stress intensity factors of cracked plates with different geometries and under various loading conditions and boundary conditions. Among a number of numerical methods found in the literature, such as finite element, boundary element and finite difference methods, the finite element method is a dominant approach to determine the stress intensity factors. Tong and Pian [1] pointed out that, in order to improve the rate of convergence of the finite element solutions to problems with stress singularities, the interpolation functions must include terms that can account for the analytical form of the singularity. In the last two decades, element-free-based methods, which are very similar to the MLS-Ritz method used herein, have been proposed for finding the stress intensity factors because they can easily include the functions that correctly describe the singular behaviors at the tip of a crack [2–7].

Numerous studies have been conducted for vibration analyses of side-cracked plates. Based on classical plate theory, integral equation techniques [8–12] have been applied to analyze a rectangular plate with two opposite edges or four edges simply supported. To consider the vibrations of a cracked rectangular plate with arbitrary boundary conditions, finite element methods [13,14], an extended finite element method [15], and the Ritz method [16–20] have been also used.

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Nomenclature

a, b	plate lengths in x-and y-directions, respectively
c_y	the distance between the x-axis and the intersection point of a crack and $x=a$
d	crack length
d_{m_uv}	the support of weight function for in-plane displacement functions
d_{m_w}	the support of weight function for transverse displacement function
D	flexural rigidity of plate ($=Eh^3/12(1-\upsilon^2)$)
E	Young's modulus of plate
f_x, f_y	the external tractions in the x and y directions, respectively
h	plate thickness
$N_{xx}^{(0)}, N_{yy}^{(0)}, N_{xy}^{(0)}$	initial membrane and shearing forces (per unit length of the plate) in the corresponding directions
N_d	a parameter to define the distance between two adjacent nodes
$\bar{N}$	the buckling intensity of normal force ($=\bar{\sigma}h$)
$\bar{N}_{cr\_d/a=0}$	the critical buckling load of an intact plate
$\hat{N}$	non-dimensional buckling loads ($=\bar{N}a^2/(\pi^2D)$)
$\hat{N}_{cr}$	non-dimensional critical buckling load
u,v	displacement components in the x and y directions due to preloading, respectively
$\bar{w}$	the magnitude of the transverse harmonic vibrations of a plate
α	inclined angle of crack
β_p	a loading intensity ($=\bar{N}/\bar{N}_{cr\_d/a=0}$)
υ	Poisson's ratio
ρ	mass per unit volume
ω	angular frequency
Ω	non-dimensional frequency parameter ($=\omega a^2\sqrt{\rho h/D}$)
$\sigma_{xx}^{(0)}$, $\sigma_{yy}^{(0)}$, $\sigma_{xy}^{(0)}$	initial in-plane stress components due to preloading
$\bar{\sigma}$	the prescribed intensity of traction on the boundaries
Δ^-, Δ^+	the ratios of the critical compressive and tensile buckling loads of a cracked plate to the critical buckling load of the plate without the crack

The buckling behaviors of cracked rectangular plates have also been investigated by many researchers. Most of the studies applied finite element methods and considered a plate with an internal crack [21–25]. Based on the classical plate theory, Markström and Storåkers [26] applied elements, which have continuity of corner displacements and slopes at the mid-point of interelement boundaries, to determine buckling loads of a rectangular plate with one or two horizontal edge cracks under uniform uniaxial tensile loads. Alinia et al. [27] performed buckling analysis of a simply supported, side-cracked, rectangular shear panel via the commercial finite element program ANSYS. Based on a shear deformable plate theory, Khedmati et al. [28] utilized an eight–node isoparametric quadratic plate element to determine buckling loads of a rectangular side-cracked plate with simple supports and subjected to uniaxial uniform compression. The numerical results in the aforementioned studies were presented in graphics. Because the thickness of plate was specified for the published results obtained from finite element approaches, doubt has arisen whether the classical plate theory was rigorously applied for those results. Other than various finite element approaches, integral equation method [9,29] and the Ritz method [30] have also been utilized. However, these studies assumed the uniform distribution of internal stress resultants in a cracked plate, which is not realistic, and no tensile buckling loads were obtained.

Although much research has been conducted on vibrations of loaded rectangular intact plates (see the review articles [31–33]) or plates with interior cutouts [34,35], to the best knowledge of the authors, there are no studies on vibrations of a loaded rectangular plate with a side crack.

Due to its high versatility and conceptual simplicity, the Ritz method is widely used to determine approximately the modal vibration characteristics of plates or the buckling loads of plates with internal stress resultants explicitly described by functions. The main purpose of the work is to present accurate solutions for vibrations and stability of a loaded rectangular plate with a side crack via the Ritz method with the admissible functions constructed by the moving least squares approach, which is termed the MLS-Ritz method. An admissible function established by the moving least square approach has a compact support and good ability of describing local behaviors of physical quantities of interest.

Because crack buckling behavior is significantly affected by the in-plane stress distribution around a crack tip, to get more accurate results, crack functions that show the exact singular order of in-plane stresses and discontinuity of in-plane displacement across the crack are involved in basis functions for constructing the admissible functions. Furthermore, another set of crack functions for the out-of-plane displacement is also employed to appropriately describe the stress singularities of bending moments and transverse shear forces at the crack tip and the discontinuity of displacement and slope across the crack.

Comprehensive convergence studies are performed for the stress intensity factor, buckling loads and vibration frequencies of a cracked rectangular plate under uniform in-plane loading at its two opposite edges. The validity of the present approach for determining the initial stresses is demonstrated by comparing the present stress intensity factors with the published ones, while the correctness of the present buckling loads is confirmed by comparing with those obtained by commercial finite element package ABAQUS. The present vibration frequencies of a cracked plate with no loading are also compared with the published ones. Finally, the present approach is applied to investigate the effects of location, length and orientation of side crack on the vibration and stability behaviors of cracked rectangular plates with four simply supported edges or with two opposite edges simply supported and free on the other edges. Numerical results are presented for the first two compressive buckling loads and the first tensile buckling load and the first five frequencies, most of which have not been seen in the previous literature.

2. Methodology

2.1. Problem formulation

To investigate the free vibrations of a loaded plate using classical plate theory and the Ritz method, the total energy of the system is defined as

$$\Pi=\frac{D}{2}\iint_A\{(\bar{w}_{,xx}+\bar{w}_{,yy})^2-2(1-\upsilon)[\bar{w}_{,xx}\bar{w}_{,yy}-(\bar{w}_{,xy})^2]\}dA+\frac{1}{2}\iint_A N_{xx}^{(0)}\bar{w}_{,x}^2+2N_{xy}^{(0)}\bar{w}_{,x}\bar{w}_{,y}+N_{yy}^{(0)}\bar{w}_{,y}^2\,dA-\frac{\rho h\omega^2}{2}\iint_A\bar{w}^2dA, \quad (1)$$

where $\bar{w}$ is the magnitude of the transverse harmonic vibrations of a plate with angular frequency ω; D, h, υ and ρ are the flexural