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Initial imperfection effects on postbuckling response of laminated plates under end-shortening strain using Chebyshev techniques

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ABSTRACT

The effects of initial imperfection on postbuckling behaviour of laminated plates subject to end shortening stain are investigated in this paper. Different boundary conditions and lay-up configurations are considered and classical laminated plate theory is used for developing the equilibrium equations. The equilibrium equations are solved directly by substituting the displacement fields with equivalent finite double Chebyshev polynomials. This technique allows imposing different combinations of boundary conditions on all edges of composite laminated plates. The final nonlinear system of equations is obtained by discretizing both equilibrium equations and boundary conditions with finite Chebyshev polynomials. Nonlinear terms caused by the product of variables are linearized by using quadratic extrapolation technique to solve the system of equations. Since number of equations is always more than the number of unknown parameters, the least squares technique is used to solve the system of equations. Some results for angle-ply and cross-ply composite plates with different boundary conditions are computed and compared with those available in the literature, wherever possible.

1. Introduction

Structures made of laminated composite material are increasingly used in aerospace, automotive, marine and other engineering applications. The high strength and stiffness properties along with low weight, good corrosion resistance, enhanced fatigue life and low thermal expansion are the most well-known characteristics of composite materials. These advantages along with the increasing usage of these materials stimulate the advancement in analysis of laminated beams, plates and shells. Many researches have been carried out on beams, plates and shells with composite materials and also functionally graded materials. In order to fully exploit the lightweight potential of such structures, it is of high practical importance to consider load ranges beyond bifurcational buckling. They often have significant and unavoidable initial geometrical imperfections, so considering the imperfection is necessary. Therefore, the investigation of initial imperfection on postbuckling behaviour of these structures are also very important.

Buckling and postbuckling behaviour of laminated composite plates was considered by many researchers in the past. Turvey and Marshall [1] and Argyris and Tenek [2] presented excellent reviews on methods investigating buckling and postbuckling behaviour of structures.

Loughlan [3, 4] investigated the effects of local buckling and post-local buckling mechanism on the axial stiffness and failure of uniformly compressed isotropic I-section and box-section struts. They obtained complete loading history of the compression struts from the onset of elastic local buckling through the nonlinear elastic and elasto-plastic postbuckling phases of behaviour to final collapse and unloading. Dawe et.al. [5] employed semi-analytical finite strip method (FSM) to investigate postbuckling behaviour of composite structures under end-shortening. The finite strip method can be considered as a kind of finite element method in which a special element called strip is used. The basic philosophy is to discretize the structures into longitudinal strips and interpolate the behaviour in the longitudinal direction by different functions, depending on different versions of FSM and in the transverse direction by polynomial functions. Wang and Dawe [6] developed a spline finite strip method on studying relatively thick composite plates using First order Shear Deformation plate Theory (FSDT). Ovesy et.al. [7] developed two finite strip methods for predicting the geometrically non-linear response of rectangular thin plates with simply supported ends when subjected to uniform end shortening in their plane. They [8–10] also employed different versions of finite strip methods to predict postbuckling behaviour of box and channel sections under end shortening. Stamatelos et. al. [11] developed a methodology for the analytical assessment of local buckling and post-buckling behaviour of isotropic and

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orthotropic stiffened plates. In their approach, it is assumed that the stiffened panel segment is located between two stiffeners, while the remaining panel is replaced by equivalent transverse and rotational springs of varying stiffness, which act as elastic edge supports. Ghannadpour et.al. introduced a full analytical finite strip method to calculate the relative postbuckling stiffness of I-section and Box-section struts [12,13]. Ovesy et.al. [14] also evaluated the buckling and post-buckling behaviour of delaminated composite plates with multiple through-the-width delaminations. They handled both local buckling of delaminated sub-laminates and global buckling of the whole plate. Lui and Lam [15] employed a finite strip method for predicting response of laminated plates with initial imperfection in very general shape when subjected to progressive end shortening. Zou and Lam [16] developed procedure from CLPT formulation to higher order plate theory in order to calculate postbuckling behaviour of thick plates. Ovesy et al. [17,18] used both spline and semi-analytical finite strip methods for predicting the postbuckling response of rectangular composite laminated plates with initial imperfections, when subjected to progressive end shortening.

In other researches, Shen et al. [19,20] investigated the post-buckling analyses of composite and functionally graded plates under thermal and mechanical loads. He also [21,22] analysed simply supported plates with initial imperfection in thermal environment. These studies are mostly limited to the selection of simply supported boundary conditions on all edges of the plates.

However, in some methods like spectral methods, the mathematical polynomials are used to estimate the displacement fields. These polynomials allow one to analyse the plates with combination of different boundary conditions on all edges. Chebyshev polynomials are one of these powerful mathematical polynomials with useful properties that can help to predict the plate's behaviour.

For the first time, Alwar and Nath [23] used Chebyshev polynomials to solve equilibrium equations and analysed the nonlinear behaviour of isotropic circular plates. They obtained the solution of the differential equation as a sum of the Chebyshev polynomials. Nath and Alwar [24] extended their method to the rectangular domain based on classical plate theory. In circular domains, univariate Chebyshev polynomials could be used to solve equilibrium equations while for rectangular domains, bivariate Chebyshev polynomials are necessary. Shukla and Nath [25] have studied large deflections of moderately thick composite plates with different lay-up and various boundary conditions under lateral pressure. They [26] also analysed buckling and post-buckling of perfectly flat angle-ply laminated plates subjected to combined in-plane mechanical load and temperature gradient across the thickness. While in this paper, the effects of initial imperfection on postbuckling behaviour of laminated plates subject to end shortening stain are investigated for different boundary conditions and lay-up configurations. By substituting the displacement fields with equivalent finite double Chebyshev polynomials, the equilibrium equations are solved directly. Using this method allows one to analyse the composite laminated plates with combination of different boundary conditions on all edges. The final nonlinear system of equations is obtained by discretizing both equilibrium equations and boundary conditions with finite Chebyshev. The least squares technique is used to solve the system of equations since number of equations is always more than the number of unknown parameters. Some results for angle-ply and cross-ply composite plates with different boundary conditions are computed and compared with those available in the literature, wherever possible.

2. Theoretical formulation

Fig. 1 shows a typical rectangular imperfect plat in arbitrary coordinate. The plate is made of laminated composite material. Classical laminated plate theory (CLPT) is used to form the equilibrium equations of plates.

These equations are shown as [27]

$$\begin{aligned}
&\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=0\\
&\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+\mathrm{N}(\bar{w})=0
\end{aligned}\tag{1}$$

Where M and N are the resultant forces and moments respectively and $\mathrm{N}(\overline{w})$ is defined as [27]:

$$\mathrm{N}(\overline{w})=\frac{\partial}{\partial x}\left(N_{xx}\frac{\partial\overline{w}}{\partial x}+N_{xy}\frac{\partial\overline{w}}{\partial y}\right)+\frac{\partial}{\partial y}\left(N_{xy}\frac{\partial\overline{w}}{\partial x}+N_{yy}\frac{\partial\overline{w}}{\partial y}\right).$$

To solve the above equations, the displacement fields based on CLPT are defined as:

$$\begin{aligned}
&\bar{u}(x,y,z)=u(x,y)-z\frac{\partial w(x,y)}{\partial x}\\
&\bar{v}(x,y,z)=v(x,y)-z\frac{\partial w(x,y)}{\partial y}\\
&\overline{w}(x,y,z)=w(x,y)
\end{aligned}\tag{2}$$

Where $\bar{u}$, $\bar{v}$, $\overline{w}$ are components of displacement in the x, y and z directions at a general point, respectively, whilst u, v, w are defined at the middle surface of the plates ($z=0$).

Substitution of the displacement fields, Eq. (2), in the green's expression for nonlinear strains with usual Von-Karman assumptions leads to the three strain components for a perfect plate as [27]:

$$\bar{\boldsymbol{\varepsilon}}=\begin{Bmatrix}\bar{\varepsilon}_{xx}\\ \bar{\varepsilon}_{yy}\\ \bar{\gamma}_{xy}\end{Bmatrix}=\boldsymbol{\varepsilon}-z\boldsymbol{\kappa}=\begin{Bmatrix}\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\\ \frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\end{Bmatrix}-z\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix}\tag{3}$$

Where $\bar{\varepsilon}_{xx}$ and $\bar{\varepsilon}_{yy}$ are axial strains, $\bar{\gamma}_{xy}$ is shear strain and $\boldsymbol{\varepsilon}$ and $\boldsymbol{\kappa}$ are membrane and flexural strain vectors, respectively.

If w^* is assumed as small initial imperfection, the net strain components in the middle surface of the imperfect plate become [16]

$$\begin{Bmatrix}\bar{\varepsilon}_{xx}\\ \bar{\varepsilon}_{yy}\\ \bar{\gamma}_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w^*}{\partial x}\right)\\ \frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial w^*}{\partial y}\right)\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)+\left(\frac{\partial w}{\partial y}\frac{\partial w^*}{\partial x}\right)+\left(\frac{\partial w}{\partial x}\frac{\partial w^*}{\partial y}\right)\end{Bmatrix}-z\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix}\tag{4}$$