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Optimal design of composite thin-walled beams using simulated annealing

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ABSTRACT

In this paper, a problem formulation and solution methodology for optimal design of thin-walled composite beams is presented. The aim is to maximize the buckling load capacity and minimize the weight of the beam. For this purpose, a theoretical model is developed for the analysis of thin-walled composite beams under a state of arbitrary initial stresses. In order to find the optimal solution, Simulated Annealing method is implemented. Design variables are taken as the stacking sequences of laminate and the dimensions of the cross-section. The space of feasible solutions is constrained by strength, displacements, global and local buckling and geometric conditions.

1. Introduction

Composite materials are mostly used in applications where stiffness-to-weight or strength-to-weight ratios are critical. The design of composite structures is a difficult task due to the numerous design variables which have to be simultaneously taken into account. For example varying fiber orientation in each ply, or in a certain number of plies, can produce a large number of acceptable designs that support a specific loading condition. Hence it is necessary to find the structure with the best configuration for a specific application. This can be achieved through a process of optimization design. In the last years many works have been developed related to the optimization of composite structures [1–4]. In addition, another challenge to face is represented by the fact that in engineering practice the design variables are not continuous. Thus, design variables can only take values from a predefined discrete set. Therefore, in order to find the absolute optimum of an objective function a global optimization method must be employed. Stochastic optimization techniques are suitable for this propose, because they can search into a large solution space and escape from local optimum points.

The design of thin-walled composite beams is a very important topic in construction engineering. One-dimensional models are suitable for optimization problems since they capture the main features of the structure behavior, while they are simple enough to be employed in extensive computation. Many one-dimensional models have been developed and implemented [5–9]. A very important aspect in the behavior of composite beams is the shear flexibility due to non-uniform torsional warping. This has been considered in a complete form in several works [10–13]. Additionally, the local buckling must be regarded on the design [14,15]. The local buckling analysis of composite beams is accomplished by modeling the flanges and the webs individually, considering the flexibility of the flange-web connections. Qiao and Shan developed analytical predictions for local buckling of some FRP (Fiber Reinforced Plastic) beams considering the elastic restraints of the flange-web connections [16]. Tarján et al. extended this work to include other boundary conditions and analyzed local buckling analysis of orthotropic composite beams [17].

In the present study, a solution methodology for minimum weight and maximum global buckling load of orthotropic thin-walled beams, with open or closed cross-sections, is developed. The corresponding design variables are given by the dimensions of the cross-section, the thickness of each layer, the fiber orientation of each layer and the number of layers of the laminate. The set of constraints includes global and local stability conditions, strength condition and technological and constructional requirements in the form of geometric relations. A theoretical model for the stability analysis of composite thin-walled beams is proposed to find these conditions. This model incorporates in a full form the shear

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flexibility (bending and non-uniform warping effects) and takes into account an arbitrary state of initial stresses. Also, a finite element with two nodes and seven degrees-of-freedom per node is employed to solve the governing equations. Moreover, analytical solutions of the problem of local buckling of orthotropic beams subjected to linearly varying axial loads are employed to impose a local buckling condition. Finally, Simulated Annealing method is applied to find the optimal fiber orientation of the laminates and the cross-section dimensions.

2. Optimal design

In order to find the optimal design which satisfies specific structural conditions a mathematical model of the optimization problem is proposed. The aim is to maximize the global buckling load and minimize the weight of the beam at the same time. To carry out both targets, a dimensionless objective function is assumed to be in the following form [18]

$$F(\mathbf{x}) = \frac{E_1 A^{3/2}}{M_{cr}}, \tag{1}$$

where $\mathbf{x}$ is the vector of the design variables, A is the cross-sectional area, E_1 is the Young's modulus in the x_1 direction and M_{cr} is the critical moment given by

$$M_{cr} = \lambda M_0^{ref}, \tag{2}$$

being M_0^{ref} a reference moment, which is function of the current loading, and λ a dimensionless global load parameter. Multiplying λ by the current loading, the critical load is obtained and the global elastic instability of the structure is achieved.

The components of $\mathbf{x}$ are defined as: the fiber orientation of the kth layer θ_k, the dimensions of the cross-section, b and h, the thickness of each layer t_c and the number of layers of the laminate n. The magnitudes t_c and n define the total wall thickness t (see Fig. 1).

The structural constraints take into account the strength of laminate, the global and local stability of the beam and the maximum allowed displacement. The structure must also verify the condition of thin-walled beams. In addition, it is established that the area of the cross-section does not overcome a maximum value (A_{max}) according to the requirements of a preliminary design. In summary, the complete optimization problem is written as follows

$$\begin{aligned} &\min_{\mathbf{x} = (h,b,t_c,n,\theta_k)} \quad \frac{E_1 A(\mathbf{x})^{3/2}}{M_{cr}(\mathbf{x})}, \\ &\text{subject to} \\ &R_{min}(\mathbf{x}) > 1, \quad \lambda(\mathbf{x}) > 1, \quad \delta(\mathbf{x}) \le \delta_{máx}, \quad \lambda_L(\mathbf{x}) > 1, \\ &A(\mathbf{x}) \le A_{max}, \quad min\left(b/t,\ h/t\right) \quad \ge 10, \\ &\mathbf{x}^{\mathbf{L}} \le \mathbf{x} \le \mathbf{x}^{\mathbf{U}}, \quad \theta_k = \left(0°,\ 90°\right), \end{aligned} \tag{3}$$

where λ_L is the local load parameter and the ratios b/t and h/t represent the slenderness of the beam. The displacement module (δ) is defined as the square root of the sum of squared displacement and must be less than the maximum displacement allowed. In this case, we considered that limit as a standard value set to 2.5 per thousand of the beam length ($L/400$). R_{min} is the lowest safety factor, which is calculated at the points shown in Fig. 2. This safety factor is obtained according to the Tsai-Wu failure criterion [19]. To calculate R_{min}, the stresses are needed. These are calculated employing the beam model presented in the next section.

3. Composite thin-walled beam model

A theoretical model is developed to find displacements, stresses and global buckling loads which will be used to solve the optimization problem. This model represents an extension of the work in reference [11], in order to account an arbitrary state of initial stresses. The effect of shear deformability due to both bending and non-uniform warping is taken into account and the Hellinger-Reissner Principle is employed. Composite thin-walled beams of closed and open cross-section subjected to any boundary condition are considered. This theory is valid only for FRP materials with orthotropic lamination (symmetric balanced, orthotropic and cross-ply laminates).

3.1. Displacement and strain fields

In the Fig. 3, B is a generic point in the middle line of the wall. Two reference points are employed: the point C, coincident with

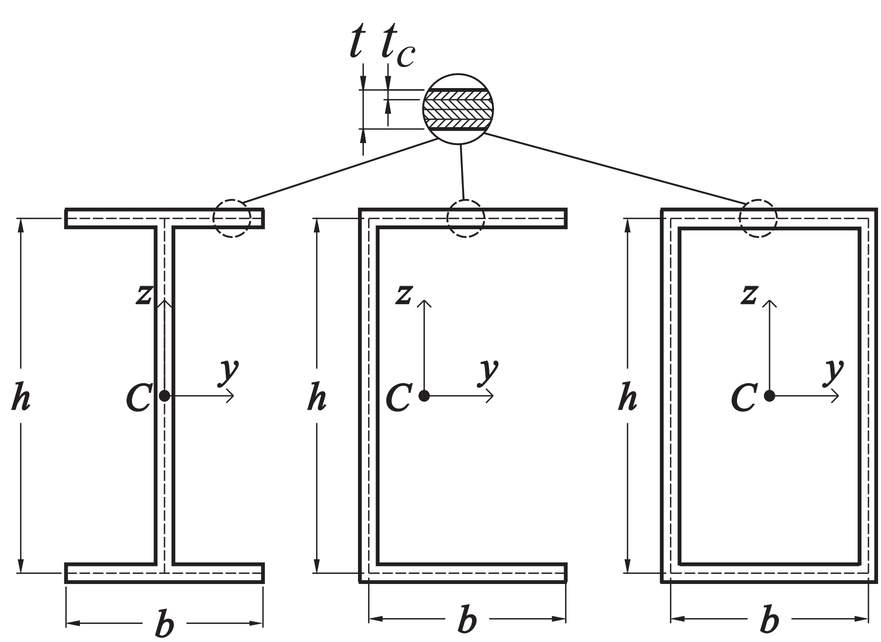

Fig. 1. Detail of the dimensions of I, C and box profiles.