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Transient analysis of rotating functionally graded truncated conical shells based on the Lord–Shulman model

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A B S T R A C T

The generalized coupled thermoelasticity based on the Lord–Shulman (L-S) theory is employed to study the transient thermoelastic behavior of rotating functionally graded (FG) truncated conical shells subjected to thermal shock with different boundary conditions. Material properties are assumed to be graded in the thickness direction, which it can vary according to a simple power law distribution. The governing equations together with related boundary conditions are discretized using a mapping-differential quadrature method (DQM) and Newmark time integration scheme in the spatial and temporal domains, respectively. The formulation and method of the solution are validated by showing their fast rate of convergence and comparison with available data in the open literature. Then, the effects of the material graded index, the angular velocity, semi-vertex angle together with other geometrical parameters on the thermal behavior of the rotating FG truncated conical shells under thermal shock, including temperature change, displacement and stress components are investigated.

1. Introduction

Conical shells are being used as structural components in modern industries of different engineering fields such as aerospace and ship structures, pressure vessels and piping, for instance, roofs for tank, as reducers in piping, end closures for pressure vessels and liquid storage tanks. Some parts of aircraft and rockets are required to work under radiation and high temperatures and which cause heterogeneity in the material properties. These applications of conical shells in different industry demands that the heterogeneity of the materials should be considered for the thermoelastic analysis of the shell.

Functionally graded materials (FGMs) are a relatively new type of material in which the elastic and thermal properties change from one surface to the other, gradually and continuously. These materials are usually made of a mixture of ceramic and metal. The advantage of using functionally graded materials is that they are able to withstand high-temperature gradient environments while maintaining their structural integrity. FGMs are mainly used in high temperature environments. Currently, rotating functionally graded (FG) truncated conical shells, especially under a high-temperature environment, have found wide applications as structural members in modern industries such as aviation, aerospace and mechanical engineering. Therefore, for the purpose of their engineering design and manufacture, the transient analysis is essential for accurate evaluation of the induced stresses, deformations and temperature under these service conditions.

In most thermoelasticity problems, the equation of heat conduction in the classical uncoupled theory of thermoelasticity is a parabolic equation which yields an infinite speed of heat waves propagation. Furthermore, the equation of heat conduction in this theory is independent of any elastic terms [1]. However, based on the classical uncoupled theory, these two phenomena are not consistent with physical observations. It is well known that classical diffusion theory may break down when the time rate of change of thermal boundary conditions, or other imposed thermal loads, are large enough to excite the thermal stress wave propagation. Then, the coupled theory of thermoelasticity becomes trustworthy in describing the diffusion process and predicting the stress and temperature distribution. In this theory, the field of temperature is coupled with the field of displacement and therefore, any attempt to define the temperature distribution within the body should be done with simultaneous consideration of thermoelastic equations.

The thermoelastic analysis of non-rotating FG cylindrical shells based on the classical uncoupled theory of thermoelasticity has been investigated by many researchers; see for example [2–7]. In comparison with the research works in these topics, only limited studies can be found on the coupled thermoelasticity of stationary FG cylindrical shells [8–14]. In these interesting studies [8–14], the coupled thermoelastic responses of functionally graded cylindrical shells subjected to the thermal load with prescribed temperature

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or convection on the external surfaces of FG shells were presented. Also, in all of these studies, the effective material properties were obtained using the power law distribution. On the other hand, there exist some investigations concerned with the thermal analysis of rotating FG cylindrical shells [15–21].

However, to the best of the authors' knowledge, the only available study related to the thermoelastic analysis of FG truncated conical shell under transient thermal loads is limited to those of the stationary FG shell [22]. Transient thermal stresses in thick truncated cones with both edges clamped using the classical theory of thermoelasticity in conjunction with the graded finite element method (GFEM) based on Rayleighe-Ritz energy formulation was presented by Asemi et al. [22]. The Cranke-Nicolson algorithm was used to solve the problem in the temporal domain. They assumed that the material properties vary in the thickness direction according to a power law. It should be mentioned that there are some research works which are concerned with the dynamic and free vibration analyses of stationary and rotating FG truncated conical shells [23–30].

A review of literature reveals that the transient analysis of rotating FG truncated conical shells subjected to thermal shock based on the coupled theory of thermoelasticity is not investigated yet. Therefore, this problem is studied in this paper due to its practical importance. Material properties of the shell are assumed to be graded in the thickness direction. The solution procedure includes transformation of the governing equations from the physical domain to computational domain and then discretization of the spatial and temporal derivatives by employing the differential quadrature method as an efficient and accurate numerical tool [6,18,25–29,31,32,33] in conjunction with the Newmark time integration scheme [34], respectively. After validating the method of solution and showing the fast rate of convergence and high accuracy of the technique, parametric studies are carried out to exhibit the influences of material properties, angular velocity, relaxation time, length-to-mean radius and thickness ratios, semi-vertex angle and boundary conditions of the truncated conical shells on the temperature distribution, displacement and stress components and their time histories at different points.

2. Governing equations

Consider an FG truncated conical shell which is subjected to thermal shock and rotates about its axis with a constant angular velocity Ω. The truncated conical shell has the bottom inner radius R_2, top inner radius R_1, thickness h, length L, semi-vertex angle β and the mean radius of R_m at section $(L\cos\beta/2)$ as shown in Fig. 1 (a) and (b). Due to axisymmetric geometry, material properties, thermal loading and boundary conditions of the shell, an axisymmetric deformation is expected within the structure. Consequently, the cylindrical coordinate system with coordinate variables (r, z), which are shown in Fig. 1(a) and (b), is used to label the material points of the shell in the undeformed reference configuration. The displacement components of an arbitrary material point of the shell are denoted as u and w along the r and z-directions, respectively.

Material properties of the FG shell vary continuously and smoothly in the thickness direction of the shell. In this study, the effective material properties are obtained using the power law distribution. The material composition continuously varies such that the inner surface of the shell is ceramic rich, whereas the outer surface is pure metal. According to the power law distribution, a typical effective material property 'P' of the shell can be represented as,

$$P(r,z) = P_c + (P_m - P_c)\left(\frac{\bar{r}}{h\sec\beta}\right)^p \tag{1}$$

where $\bar{r}[= r - R_i(z)]$; the subscripts m and c refer to the metal and ceramic constituents, respectively; p is the power law index or the material property graded index which is a positive real number.

Based on the linear theory of elasticity, the constitutive

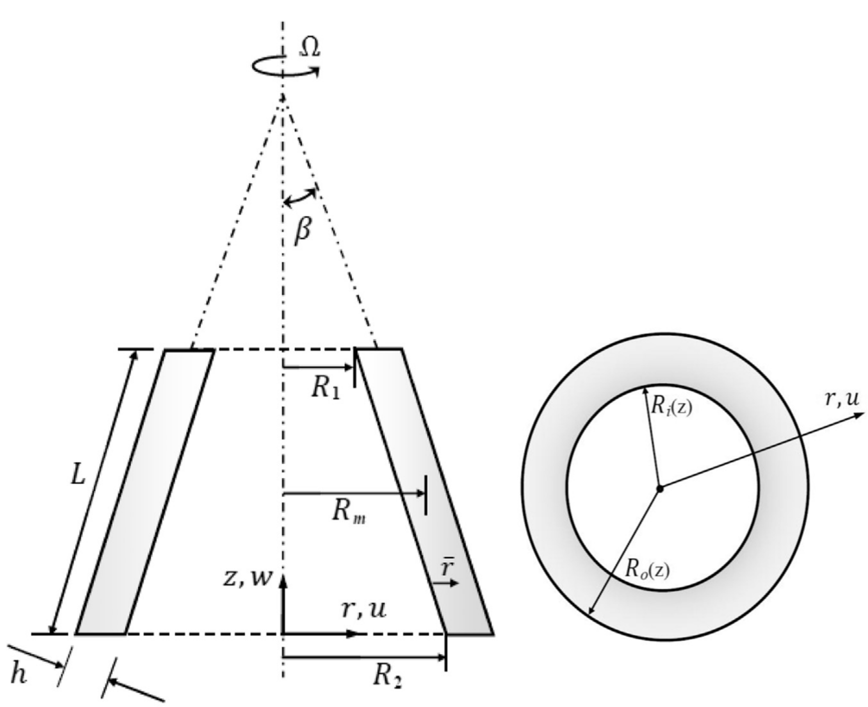

Fig. 1. (a) and (b): Geometry of the rotating FG truncated conical shells.