Thin-Walled Structures 102 (2016) 1–17

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Coupled electro-mechanical effects and the dynamic responses of functionally graded piezoelectric film-substrate circular hollow cylinders

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ARTICLE INFO

Article history:
Received 20 July 2015
Accepted 10 January 2016
Available online 21 January 2016

Keywords:
Finite layer methods
Vibration
Functionally graded piezoelectric material
Film
Substrate
Cylinders

ABSTRACT

Based on Reissner's mixed variational theorem (RMVT), we developed finite cylindrical layer methods (FCLMs) to investigate the quasi-three-dimensional (3D) dynamic responses of simply-supported, two-layered functionally graded piezoelectric material (FGPM) film-substrate circular hollow cylinders with open- and closed-circuit surface conditions. The FGPM film-substrate cylinder considered in this work consists of a thick and soft FGPM substrate with a surface-bonded thin and stiff homogeneous piezoelectric material (HPM) film. The material properties of the FGPM layer are assumed to obey an exponent-law exponentially varying with the thickness coordinate, and the piezoelectric ceramic material PZT-4 is taken to be the reference material. The accuracy and convergence rate of FCLMs with different orders are assessed by comparing their solutions with the exact 3D ones available in the literature. The convergent solutions of FCLMs for the lowest frequency parameters and their corresponding modal electric and elastic variables of the FGPM film-substrate cylinder are presented. The influences of various factors with regard to some of the key dynamic responses of the cylinder are examined, such as the coupled electro-mechanical characteristics, material-property gradient index, surface boundary conditions, aspect ratio, and film-substrate thickness ratio.

1. Introduction

Functionally graded piezoelectric materials (FGPMs) are a new class of advanced materials, and these have been used to form beam-, plate- and shell-like smart (or intelligent) structures for sensing, actuating and control purposes in micro- and nano-electro-mechanical systems, due to their electro-mechanical coupling characteristics [1–4]. For example, carbon nanotubes (CNTs) and nitride nanotubes (NNTs) have been embedded into the piezoelectric polymers, such as poly-vinylidene fluoride (PVDF) and piezoelectric ceramics PZTs, with a random or assigned function distribution to form the functionally graded (FG) carbon/nitride nanotube-reinforced composite structures [5–7] in order to enhance the gross material properties of the integrated structures. Moreover, functionally graded elastic materials (FGEMs) have also being widely used in a number of advanced industries. For example, sandwich elastic structures have been embedded with an FGEM core in order to reduce the stress concentration occurring at the face sheet-core interfaces, as well as some failure phenomena, such as matrix cracking, due to the discontinuous in-surface stresses [8–12].

Unlike the single-layered and multilayered homogeneous piezoelectric/elastic material (HPM/HEM) structures with constant material properties for each layer, the material properties of FGPM/FGEM structures can be designed to gradually and smoothly vary through the thickness coordinate, which provides some of the above-mentioned advantages in their practical applications, although making their analysis more complex. Numerous two- and three-dimensional (2D and 3D) theoretical methodologies and numerical models for HPM/HEM structures [13–18] have been extended to the analysis of FGPM/FGEM structures, such as global/layerwise first-order and higher-order shear deformation theories (FSDTs and HSDTs), the state space, modified Pagano, series expansion, perturbation methods, and some semi-analytical numerical ones. In conjunction with the principle of virtual displacement (PVD) and Carrera's unified formulation (CUF) [19], Neves et al. [20–23] developed the hyperbolic sine shear deformation theory and HSDT for the static, free vibration and buckling analyses of FGEM plates. The corresponding governing equations were derived by using the CUF, and were solved by means of the radial basis function (RBF)-based meshless

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http://dx.doi.org/10.1016/j.tws.2016.01.008
0263-8231/

collocation method. Zenkour [24] investigated the bending analysis of FGEM plates by using a refined trigonometric plate theory, in which the influences with regard to some crucial effects on the displacement and stress components of the plate were examined, such as the transverse shear deformation, aspect ratio, and volume fraction distribution effects. Fazzolari and Carrera [25] developed a hierarchical trigonometric Ritz formulation to evaluate the dynamic characteristics of FGEM doubly curved shells and sandwich shells embedded with an FGEM core, in which the corresponding governing equations were derived using the PVD and CUF. On the basis of the Reissner's mixed variational theorem (RMVT) [26–28], Cinefra et al. [29] proposed a variable kinematic model for the free vibration analysis of FGEM shells. Baferani et al. [30] studied the free vibration behaviors of FGEM thin annular sector plates resting on elastic foundation using the classical plate theory (CPT). Zhang et al. [31] studied the free vibration behaviors of rectangular composite laminates using a layerwise cubic B-spline finite strip method. Najafov et al. [32] investigated the vibration and stability behaviors of axially compressed truncated conical shells embedded with an FGEM layer and surrounded by an elastic medium by using the Donnell-type classical shell theory (CST). Tajeddini and Ohadi [33] developed a semi-analytical polynomial-Ritz method for the quasi-3D vibration analysis of FGEM thick, annular plates with variable thickness. In the above-mentioned articles, the material properties of each FGEM layer were assumed to obey either a power-law distribution in terms of the volume fractions of the constituent materials, or an exponent-law varying exponentially through the thickness coordinate. Finally, some comprehensive literature surveys with regard to the mechanical analyses of FGEM beams, plates and shells have been carried out [34–42].

Some articles with regard to the mechanical analysis of FGPM plates and shells are available in the literature, in which the coupling electro-mechanical effect on deformations and stresses induced in the loaded FGPM structures, and the lowest frequency and critical load parameters of FGPM ones, were mainly examined using 3D state space method [43–45], 3D perturbation method [46–50], semi-analytical differential quadrature (DQ) method [51,52], semi-analytical B-spline finite strip method [53], semi-analytical series expansion method [54], semi-analytical finite element method [55,56], CUF [57], global CPT [58], global HSDT [59], layerwise HSDT [60] and four-variable refined plate theory [61]. Some comprehensive literature surveys with regard to the mechanical analyses of FGPM structures have been conducted [62,63]. After a close literature review, we found that there are relatively few articles with regard to the RMVT-based quasi-3D coupled analysis of two-layered FGPM film-substrate cylinders. Most of the above-mentioned articles apply the PVD-based 2D piezoelectric structure theories, rather than the RMVT-based ones, and deal with the mechanical behaviors of single-layered and sandwich FGPM plates and shells [64,65], rather than two-layered FGPM film-substrate cylinders.

Within the framework of 3D elasticity, Wu and Li [66] and Wu and Chang [67] proposed the unified formulations of RMVT-based finite rectangular and cylindrical layer methods (FRLMs and FCLMs) for the quasi-3D mechanical analyses of simply-supported, single-layered and sandwich FGEM plates and cylinders, respectively. Subsequently, Wu and Li [68,69] developed the unified formulations of RMVT-based finite rectangular and cylindrical prism methods (FRPMs and FCPMs) for FGEM structures with various boundary edge conditions. It has been shown that these finite layer methods are accurate and have a fast convergence rate, that the orders used for expansion of the normal displacement and transverse normal stress components through the thickness coordinate should be identical to each other to ensure a numerical stability, and that the solutions obtained by using the same orders for the primary variables are more computationally efficient than those obtained by using the orders of primary variables different from one another. Due to the above-mentioned benefits of these finite layer methods for FGEM structures, we thus extend the unified formulation for these to that for FGPM ones in this article, accounting for the coupled electro-mechanical effects. The newly developed formulation will be applied to the dynamic responses of simply-supported, two-layered FGPM film-substrate circular hollow cylinders with open- and closed-circuit surface conditions, in which the orders used for expansions of the elastic variables through the thickness coordinate remain the same values, while the orders are variable for the electric variables. The influences with regard to some crucial effects on the lowest frequency parameters and their corresponding through-thickness distributions of modal variables are examined, such as the effects with regard to the electro-mechanical coupling characteristics, material-property gradient index, surface conditions, aspect ratio and film-substrate thickness ratio.

2. Finite cylindrical layer methods

2.1. Kinematic and kinetic assumptions

We consider a simply supported, two-layered FGPM film-substrate circular hollow cylinder, as shown in Fig. 1, in which a global cylindrical coordinate system, with x, θ and r coordinates, is located on the center of the cylinder, and h, R and L denote the total thickness, mid-surface radius and length of the cylinder, respectively. In the implementation of the developed FCLMs, the cylinder will be artificially divided into N_l cylindrical layers, the thicknesses of which are h_m $(m = 1, 2, \cdots, N_l)$, while $h = \sum_{m=1}^{N_l} h_m$. A set of local thickness coordinates, z_m $(m = 1, 2, 3, \cdots, N_l)$, is located at the mid-surface of each individual cylindrical layer, as shown in Fig. 2. The relationship among the global and local thickness coordinates and the radial one in the m^{th}-layer is $r = R + \zeta$ and $\zeta = \bar{\zeta}_m + z_m$, in which $\bar{\zeta}_m = (\zeta_m + \zeta_{m-1})/2$, and ζ_m and ζ_{m-1} are the global thickness coordinates measured from the mid-surface of

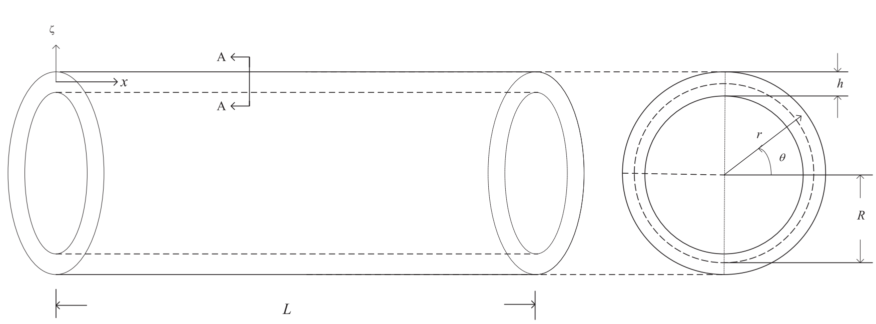

Fig. 1. The configuration, coordinates and geometric parameters of a cylinder.