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Theoretical prediction and optimization of multi-cell hexagonal tubes under axial crashing

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ABSTRACT

In this paper, the analytical formulas of mean crashing force for four different hexagonal tubes with multiple cells were first derived based on the Simplified Super Folding Element (SSFE) theory through several typical constituent elements: corner element, three-panel angular element I and three-panel angular element II. The numerical simulations of hexagonal multi-cell configurations were then correlated with the derived analytical solutions. Finally, both analytical formulas and finite element analysis (FEA) based surrogate models were employed to optimize the cross-sectional dimensions of the hexagonal tubes. From the optimization results, web-to-web (W2W) is the most efficient configuration in improving the crashing behavior, while corner-to-corner (C2C) is the worst of these four configurations. Importantly, the Pareto fronts obtained from the analytical formulas agree well with those from the FEA based surrogate models. As a result, analytical formulas could be recommended in crashworthiness optimization for the sake of computational efficiency.

1. Introduction

Over the recent years, thin-walled structures with multiple cells have been studied extensively as energy absorbers to alleviate occupants' injuries during accidents passively in vehicle engineering. For example, Wierzbicki and Abramowicz [1] pointed out that the corners on the cross-section of the tubes have an great effect on the energy absorption. Kim [2] proposed a new multi-cell configuration with four square cells located at the corners, which exhibited certain advantages in energy absorption over the conventional structures. Zhang et al. [3] demonstrated that the energy absorption of 3×3 multi-cell tube is 50% higher than the single-cell tube. Hou et al. [4] revealed that the quadruple-cell column produces higher specific energy absorption (SEA) than the single, double and triple cell columns. Rais-Rohani [5] studied energy absorption characteristics of thin wall tubes with different multi-cell and multi-corner configurations under the axial load condition. Song and Guo [6] found that windowed and multi-cell tubes have higher mean crushing forces than conventional tubes under axial and oblique loading conditions. These studies demonstrated that the multi-cell columns could have preferable mechanical behaviors over the single-cell counterparts under static and dynamic loading conditions.

However, the theoretical prediction for multi-cell tubes remains quite challenging due to their configurational complexity, which is typically constructed in different constituent elements. Chen and Wierzbicki [7] derived the analytical solutions for the mean crushing forces of multi-cell sections, foam-filled double-cell and triple-cell columns, which agreed well with the simulation results. Zhang and Zhang [8] studied the crush behaviors of different multi-cell sections under the axial load and they improved the Chen and Wierzbicki's formula of mean crushing force for multi-cell tubes. Bai et al. [9] proposed a new analytical model to predict the mean crushing force of hexagonal multi-cell thin-wall structures under the quasi-static loading by validating through experimental test and literature data. Nia and Parsapour [10, 11] revised Zhang and Zhang's formula for the tubes with unequal cell size. Recently, the crush resistances for different types of constituent elements were studied by Zhang and co-workers [8, 12–16]. Theoretical models for the corner element, three-panel element, X-shaped element and other elements with different angles and edge connectivity were established after being validated by the experimental tests.

To achieve a better performance, structural optimization has been other key area in crashworthiness design. At the early stage, researchers directly incorporated finite element analysis (FEA) in optimization algorithms, which often requires enormous

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computational cost for iterative nonlinear simulations. As an alternative, surrogate modeling later demonstrated considerable advantages over the direct method for the time-consuming nonlinear FEA-based optimization. For example, Fang et al. applied metamodeling techniques to optimize the cross-sectional configurations for multi-cell square tubes [17] and the gradient parameters for functionally graded tubes [18]. Qiu et al. [19] optimized the cross-sectional parameters based on Kriging modeling under multiple load conditions. Yang and Qi [20] also utilized Kriging modeling to optimize the empty and foam-filled square columns.

In addition, analytical formulas-based optimization has also been employed for structural crashworthiness. For instance, Kim [2] conducted an optimization using the derived analytical formula of specific energy absorption. Chen [21] derived the theoretical solutions of energy absorption for torsional resistance and critical twisting rotation and used them to optimize the foam-filled section. Hanssen et al. [22] performed the optimization based on the analytical formulas of average crush force, maximum force and stroke efficiency. Kim et al. [23] used the analytical expression as the response function during the optimization procedure. Compared to the surrogate based optimization, analytical formulas-based optimization can largely improve the computational efficiency as it no longer requires time-consuming FEA. Nevertheless, few studies have compared these two methods and it remains unknown whether the optimization results from them can agree with each other.

This study aims to explore the crashing behaviors of multi-cell hexagonal thin-wall tubes under the axial dynamic loading by using theoretical predictions and design optimization. The remainder of this paper will be organized as follows. Following the introduction, Section 2 derives the theoretical formulas of the mean crashing force for the four types of multi-cell hexagonal tubes based on the SSFE theory. Section 3 compares the analytical formulas with surrogate models in crashworthiness optimization of multi-cell tubes. Section 4 draws some conclusions.

2. Theoretical prediction of multi-cell hexagonal tubes

Although commercial FE codes are widely available to simulate the crashing behaviors of single and multi-celled tubes, the role of analytical models is irreplaceable. The analytical formula can directly estimate the crashing resistance of structure concerned without costly numerical and experimental analyses and can be very helpful in the initial stage of design for crashworthy structures.

The tube models studied in this paper are prismatic with four different cross-sectional configurations as depicted in Fig. 1. They are constituted of a hexagonal inner tube (inner wall thickness C_2=18), outer tube (outer wall thickness C_1=36) and six connecting ribs. The four different cross-sectional configurations are taken into account for a comparative study: a) corner-to-corner (C2C): the ribs connecting the inner and outer walls at the corners, b) web-to-web (W2W): the ribs connecting the inner and outer walls at the mid-wall, c) web-to-corner (W2C): ribs connecting the middle inner walls with the outer corners; d) corner-to-web (C2W): ribs connecting the inner corners with the middle outer walls. The longitudinal lengths of all these multi-cell tubes are 180 mm.

In this paper, the Simplified Super Folding Element (SSFE) method developed by Wierzbicki and Abramowicz [1] was utilized to solve the axial progressive folding of hexagonal multi-cell columns. The theory assumed that the wavelength $2H$ for each fold and wall thickness of the tube are the same.

The mean crashing resistance can be obtained based on a basic folding element consisting of some extensional and compressional elements and three stationary hinge lines (as shown in Fig. 2). Thus the mean crashing force of the element can be derived by considering the energy conservation of the system in one folding wavelength 2 H. During a complete collapse of a single fold, the external work done by axial crash is dissipated by the bending of static hinge lines and membrane deformation:

$$2HP_m k = W_{\text{bending}} + W_{\text{membrane}} \tag{1}$$

where P_{m}, W_{bending} and $W_{\text{memebrane}}$ denote the mean crashing force, bending energy and membrane energy, respectively. In reality, the flange of the folding element can never be completely flattened as shown in Fig. 2. Therefore, k is taken as the effective crashing distance coefficient, which is calculated by dividing effective crashing distance δ (see Fig. 3c) by the wavelength $2H$, as shown in Eq. (2). The value of k can be derived from numerical and/or experimental results of different tubes.

$$k = \delta/2H \tag{2}$$

2.1. The bending energy

The bending energy W_{bending} of each fold can be determined by summing up the energy dissipation at three stationary hinge lines:

$$W_{\text{bending}} = \sum_{i=1}^{3} M_0 \psi_i L_c \tag{3}$$

where $M_0 = (1/4)\sigma_0 t^2$ denotes the fully plastic bending moment per unit width and t represents the wall thickness. σ_0 is the flow stress of material with power law hardening and can be approximately as the average value of yield stress σ_{y} and ultimate stress σ_{u}. ψ and L_{c} are the rotation angle at the hinge lines and the total length of sectional width, respectively. In this study, the flanges are assumed to be completely flattened (Fig. 3(b)) after the axial compression of $2H$ [7]. Therefore, the rotation angles ψ at the three hinge lines are $\pi/2$, π and $\pi/2$, respectively (as shown in Fig. 3d). Consequently we can have,

$$W_{bending} = 2\pi M_0 L_C \tag{4}$$

2.2. Membrane energy

In order to analyze the energy dissipation under compression, the multi-cell hexagonal sections were divided into three basic components: namely corner element, three-panel angular element

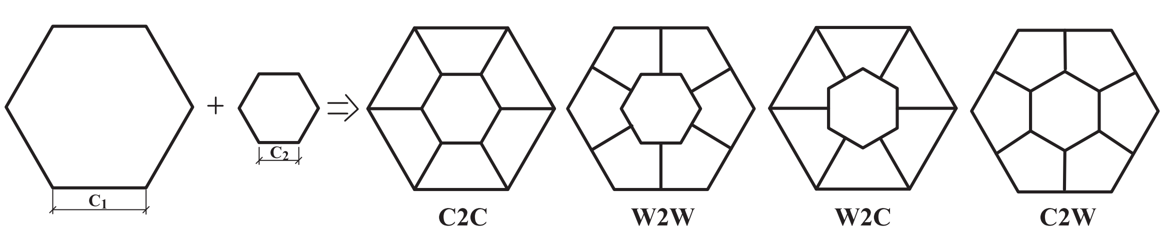

Fig. 1. Cross-sectional configurations of multi-cell, multi-corner tubes.