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Upper and lower bound solutions for lateral-torsional buckling of doubly symmetric members

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ABSTRACT

A family of three finite elements is developed for the lateral-torsional buckling analysis of thin-walled members with doubly symmetric cross-sections. The elements are based on a recently derived variational principle which incorporates shear deformation effects in conjunction with a special interpolation scheme ensuring C1 continuity. One of the elements is developed such that it consistently converges from above while another element is intended to consistently converge from below. The third element exhibits fast convergence characteristics compared to other elements but cannot be guaranteed to provide either an upper or a lower bound solution. The formulation incorporates the ability to enforce any set of linear multi-point kinematic constraints. The validity of the solution is established through comparisons with other well-established numerical solutions. The elements are then used to solve practical problems involving simply supported beams, cantilevers and continuous beams under a variety of loading conditions including concentrated loads, linear bending moments and uniformly distributed loads. The effect of lateral and torsional restraints and the location of lateral restraint along the section height on lateral-torsional buckling capacity of beams are also examined through examples.

1. Motivation

In a recent study, Wu and Mohareb [1,2] developed a shear deformable theory and finite element formulation for lateral-torsional buckling of doubly-symmetric cross-sections. The element developed was based on linear interpolation of the displacement fields, leading to a C0 continuous element, and was shown to (a) converge from above, in a manner similar to conventional finite element formulations, and (b) to exhibit particularly slow-convergence characteristics as hundreds of degrees of freedom were needed to model simple problems. Starting with the same variational principle, the present study develops an elaborate interpolation scheme leading to C1 continuity and resulting in a family of finite elements for the lateral-torsional buckling analysis of members with superior characteristics; (1) It considerably accelerates the convergence characteristics of the solution, and (2) In one of the resulting elements, discretization errors are shown to lead to lower bound estimates of the buckling capacity, a desirable feature from a design viewpoint. In the second element, they were shown to lead to an upper bound estimate while the third element exhibits the fastest convergence characteristics. The new solution is subsequently used to investigate the effect of lateral and/or torsional restraints and the effect of lateral bracing location along the web height on the lateral-torsional buckling capacity of simple and multi-span beams.

2. Literature review

Numerous studies have investigated the elastic lateral-torsional buckling (LTB) resistance of doubly-symmetric I-beams. Using the Rayleigh–Ritz method, Salvadori [3] developed the LTB solution of simply supported and continuous beams subject to a combination of axial and unequal end moments. Based on the finite difference technique, Poley [4] solved the governing buckling differential equations for cantilever beams under uniformly distributed load. Using a successive-approximation technique for solving differential equations, Austin et al. [5] developed the critical LTB solutions for beams with full torsional end restraints and partial rotational end restraints about the weak axis subjected to uniformly distributed loads and mid-span point loads. Load locations relative to the section centroid were also considered. Based on a numerical integration technique, Hartmann [6] evaluated the effect and partial lateral, torsional, and weak axis bending constraints on the LTB capacity of beams subjected to point loads, with interior supports for simply supported and continuous two-span and three-span beams. Krajcinovic [7] and Barsoum and Gallagher [8] developed a finite element for buckling analysis based on the

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Notation

A	Cross-sectional area
$[B(z)]$	Matrix relating displacement fields to integration constants
$\{C_{st}\}$	Vector of integration constants
D_{hh}, D_{xx}	Properties of cross-section related to shear deformation
d	Depth of cross-section
$\langle d(z)\rangle^T$	Field displacements
E	Modulus of elasticity
$\{F\}$	Vector of Lagrange multipliers
G	Shear modulus
$[H]$	Matrix relating nodal displacements to integration constants
I_{xx}, I_{yy}	Moments of inertia of the cross-section about x-axis and y-axis respectively
$I_{\omega\omega}$	Warping constant
J	St. Venant torsional constant
$[K_f]$	Stiffness matrix due to flexural stresses
$[K_G]_N$	Geometric matrix due to normal forces
$[K_G]_M$	Geometric matrix due to bending moments
$[K_G]_V$	Geometric matrix due to shear forces
$[K_G]_{qy}$	Geometric matrix due to load position effect of the distributed transverse load
$[K_G]_{qz}$	Geometric matrix due to load position effect of the distributed axial load
$[K_G]_{Qy}$	Geometric matrix due to load position effect of the concentrated transverse load
$[K_s]$	Stiffness matrix due to other shear stresses
$[K_{sv}]$	Stiffness matrix due to Saint Venant shear stresses
L	Length of a finite element
L_m	Span of the member
$[L(z)]$	Matrix of shape functions
m_i	Roots of quadratic eigenvalue problem
M_1, M_2	Internal bending moment at both end of an element
$M_{xp}(z)$	Strong axis bending moment as obtained from pre-buckling analysis
N_1, N_2	Internal normal forces at both ends of an element
$N_p(z)$	Resultant of the normal stresses obtained from pre-buckling analysis
$[P]$	Matrix of user-input coefficients which linearly relate any set of nodal displacements
q_y, q_z	Distributed load applied to a member acting along the y- and z-direction respectively
$[S]$	Structure elastic stiffness matrix
$[S_G]$	Structure geometric stiffness matrix
$[S_i](i = 1\ to\ 12)$	Sub-matrices needed to determine element stiffness matrices
u_b	Lateral buckling displacement
$\langle u_N\rangle^T$	Vector of nodal displacements
$\{u_s\}$	Vector of unknown displacements of the structure
U	Internal strain energy
V	Load potential energy
V_1, V_2	Internal shearing forces at both end of an element
$V_{yp}(z)$	Resultant of shear force component along y-direction obtained from pre-buckling analysis
x, y, z	Cartesian coordinates
β	End moment ratio
λ	Load multiplier
π	Total potential energy
θ_{yb}, θ_{zb}	Buckling rotation angles about y, z axes, respectively
$\omega(s)$	Warping function
ψ_b	Warping deformation (1/Length)

Vlasov thin-walled beam theory. Powel and Klingner [9] developed a thin-walled beam finite element to obtain the LTB capacity of simply supported and continuous beams subject to general loading. Their solution was applicable to doubly symmetric and mono-symmetric cross-sections. The effect of load position relative to shear center and that of lateral and torsional restraints were incorporated into the solution. Using the beam element developed by Gallagher and Padlog [10], Nethercot and Rockey [11] investigated the lateral stability of simply supported beams with discrete lateral restraints, discrete torsional restraints, and both lateral and torsional restraints, subject to uniform moments. Also, Based on the element, Nethercot [12] examined the effect of load type and lateral, torsional, and warping restraints on LTB of cantilevers and proposed expressions for the effective length of cantilevers governed by LTB. Using the same element, Nethercot [13] studied the effect of load type on LTB of simply supported beams braced laterally or torsionally under uniform moments, mid-span point load, and uniformly distributed load. Based on the finite integral method, Kitipornchai and Richter [14] studied the LTB of simply supported beams with discrete rigid intermediate translational and rotational restraints and subjected to concentrated load, end moments and uniformly distributed load. Kitipornchai et al. [15] extended their work to investigate the effect of intermediate translational and rotational discrete restraints on LTB capacity of cantilevers under uniformly distributed and concentrated loads. Based on the direct variational approach, Assadi and Roeder [16] studied the LTB of cantilevers with continuous rigid and elastic lateral restraints. Their study investigated the effects of load height, height of lateral restraint and stiffness. Based on a closed-form solution, Tong and Chen [17] investigated the LTB capacity of simply supported beams with symmetrical or mono-symmetrical I-sections, either restrained laterally or torsionally at mid-span, subjected to uniform bending moments. Wang and Nethercot [18] developed a thin-walled beam element for conducting a three-dimensional ultimate-strength analysis to assess bracing requirements for laterally unrestrained beams. They investigated simply supported I-beams with a single, three, or five equally spaced discrete torsional restraints subjected to central transverse concentrated load applied to the upper flange. Attard [19] developed solutions for estimating LTB capacity of beams with mono-symmetric and doubly symmetric sections and general boundary conditions. Albert et al. [20] developed a finite element model consisting of four-node plate elements for the web and two-node line elements for the flanges. This model predicts the LTB resistance of beams under various loading and boundary conditions while capturing distortional effects. Using this finite element model, Essa and Kennedy [21] developed effective length factors for built-in cantilevers under top and bottom flange lateral restraints and load positions relative to the shear center. Using the same element, they also developed a design approach for cantilever-suspended-span constructions [22]. Wang et al. [23] used the Rayleigh–Ritz method to determine the optimal locations for rigid lateral and torsional intermediate restraints to maximize the elastic LTB capacity of I-beams. Using the elastic buckling finite element program, BASP (Buckling Analysis of Stiffened Plates) developed by Akay et al. [24] and Choo [25], Yura [26] developed rules for bracing requirements based on the loading configuration, load height relative the shear center, location of restraint and cross-section distortion. Based on the Babnov–Galerkin method and the two-node beam finite element, Lim et al. [27] evaluated