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Some aspects of the longitudinal-transverse mode in the elastic thin-walled girder under bending moment

Zbigniew Kolakowski *, Tomasz Kubiak

Lodz University of Technology, Department of Strength of Materials (K12), 90-924 Lodz, 1/15 Stefanowskiego Street, Poland

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ABSTRACT

The present paper deals with an influence of the longitudinal-transverse mode on interactive buckling of the thin-walled composite girder with imperfections subjected to bending moment when the shear lag phenomenon and distortional deformations are taken into account. A plate model (2D) is adopted for the girder. The structure is assumed to be simply supported at the ends. A method of the modal solution to the coupled buckling problem within the first order approximation of Koiter's asymptotic theory, using the transition matrix method and Godunov's orthogonalization, has been used. The calculations have been carried out for square girders.

1. Introduction

Thin-walled structures composed of plate elements have many different buckling modes that vary in quantitative and qualitative aspects. In the case of finite displacements, different buckling modes are interrelated even with the loads close to their critical values (eigenvalues of the respective boundary problem). The investigation of stability of equilibrium states requires an application of a nonlinear theory that enables one to estimate an influence of different factors on the structure behavior. When the post-critical behavior of each individual mode is stable, their interaction can lead to unstable behavior, and thus to an increase in the imperfection sensitivity [10,13,17,18,27,30,33–35,38–40,43,44,48–50].

The concept of interactive buckling (the so-called coupled buckling) involves the general asymptotic theory of stability. Among all versions of the general nonlinear theory, Koiter's theory [24–26] of conservative systems is the most popular one, owing to its general character and development, even more so after Byskov and Hutchinson [18] formulated it in a convenient way. The theory is based on asymptotic expansions of the postbuckling path and is capable of considering nearly simultaneous buckling modes.

When components of the displacement state for the first order approximation are taken into account, this can be followed by a decrease in values of global loads. The theoretical static load carrying capacity, obtained within the frame of the asymptotic theory of the nonlinear first order approximation, is always lower than the minimum value of buckling load for the linear problem and the imperfection sensitivity can only be obtained.

Since the late 1980's, the Generalized Beam Theory (GBT) [14,15,19,20,22,47] has been developed extensively. Recently, a new approach has been proposed, i.e., the constrained Finite Strip Method (cFSM) [3,4,21,46]. These two alternative modal approaches to analyze the elastic buckling behavior have been compared in [6–8]. For the latest development directions in the GBT, see, for instance [1,2,9,16,36,37].

In the current decade, in more and more numerous publications [3–5,14,19,21,46,47], attention has been paid to the global axial mode, which is considered only in the theoretical aspect in linear issues, that is to say, under critical loads. Adany and Schafer [3] have said that *"it should be noted that this axial mode is a theoretically possible buckling mode, even though it has little practical importance"*. In the axial extension mode, longitudinal displacements of the cross-section dominate and this mode can be referred to as the shortening one (Fig. 1). The axial mode is symmetric with respect to the cross-section axis of symmetry and is symmetric with respect to the axis of overall bending.

An attempt to dispute with the statement included in the paper by Adany and Schafer [3] has been undertaken. In Ref. [31] special attention has been focused on the eigenvalue problem of the axial mode and on an interaction of the global axial extension mode with local buckling modes for a square box of the column in the first order nonlinear approximation of the perturbation method.

On the other hand, in Ref. [29] an effect of the axial mode on interactive static and dynamic buckling has been discussed for 1D and 2D models of the square thin-walled beam-column subjected to compression. In Ref. [28,32] an influence of the axial extension

* Corresponding author.
E-mail addresses: zbigniew.kolakowski@p.lodz.pl (Z. Kolakowski), tomasz.kubiak@p.lodz.pl (T. Kubiak).

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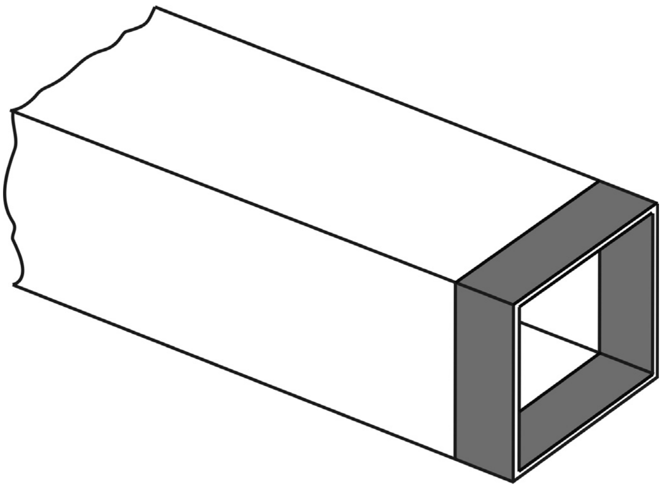

Fig. 1. Longitudinal displacement distributions of the axial mode of the beam-column.

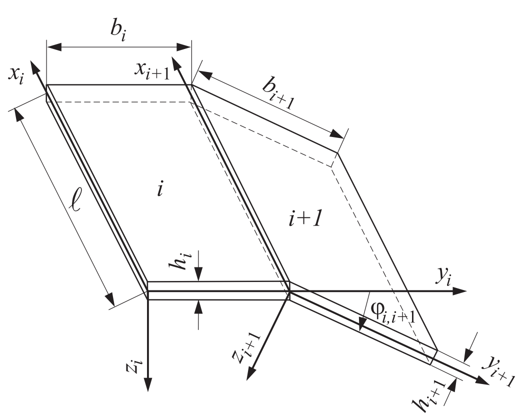

Fig. 2. Prismatic plate structure and the local co-ordinate system.

mode on interactive buckling of the beam-column has been presented. Here an effect of the longitudinal-transverse mode on interactive buckling of the isotropic and composite grinder under bending is shown. The longitudinal-transverse mode is close in nature to the axial mode in the case of short beam-columns under compression (for a more detailed analysis, see [29]). For the longitudinal-transverse mode, maximum amplitudes of the components of displacements u_{max}, v_{max}, w_{max} are close to one another.

In the present study, a plate model (2D) of the girder has been adopted to describe all buckling modes. Instead of the finite strip method, the exact transition matrix method and the numerical method of the transition matrix using Godunov's orthogonalization is used in this case. The differential equilibrium equations have been obtained from the principle of virtual works taking into account: Lagrange's description, full Green's strain tensor for thin-walled plates and the second Piola–Kirchhoff's stress tensor. The solution method assumed in this study allows for analyzing interactions of all buckling modes. The interaction between all the walls of structures being taken into account, the shear lag phenomenon and also the effect of cross-sectional distortions are included. The most important advantage of this method is such that it enables us to describe a complete range of behavior of thin-walled structures from all global to the local stability [30,33–35]. The solution method has been partially presented in Ref. [33].

In this study, special attention is focused on an influence of the longitudinal-transverse mode on coupled buckling with global and local buckling modes of isotropic and composite girders with a square cross-section.

2. Formulation of the problem

A prismatic thin-walled girder built of panels connected along longitudinal edges has been considered. The composite girder is simply supported at its ends. In order to account for all modes and coupled buckling, a plate model of the thin-walled structure has been assumed. The isotropic and composite materials the girders are made of are subject to Hooke's law.

For each plate component, precise geometrical relationships are assumed to enable the consideration of both out-of-plane and in-plane bending of the i-th plate [30,33,34]:

$$\begin{aligned} \varepsilon_x &= u_{,x} + \tfrac{1}{2}(w_{,x}^2 + v_{,x}^2 + u_{,x}^2) \\ \varepsilon_y &= v_{,y} + \tfrac{1}{2}(w_{,y}^2 + u_{,y}^2 + v_{,y}^2) \\ 2\varepsilon_{xy} &= \gamma_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y} + u_{,x}u_{,y} + v_{,x}v_{,y} \end{aligned} \tag{1}$$

and

$$\kappa_x = -w_{,xx} \quad \kappa_y = -w_{,yy} \quad \kappa_{xy} = -2w_{,xy} \tag{2}$$

where: u, v, w – components of the displacement vector of the i-th plate along the x, y, z axis direction for the local co-ordinate system (Fig. 2), respectively, and the plane $x-y$ overlaps the central plane before its buckling.

In the majority of publications devoted to stability of plate structures, the terms $(v_{,x}^2 + u_{,x}^2)$, $(u_{,y}^2 + v_{,y}^2)$ and $(u_{,x}u_{,y} + v_{,x}v_{,y})$ are neglected for $\varepsilon_x, \varepsilon_y, \gamma_{xy} = 2\varepsilon_{xy}$, correspondingly, in strain tensor components (1) according to the von Karman's nonlinear plate theory (i.e. classical plate theory CPT) [17,18,24–26,43,44,51,52]. The main limitation of the assumed theory lies in an assumption of linear relationships between curvatures (2) and second derivatives of the displacement w(discussed in detail in, e.g., [41,42]). This is

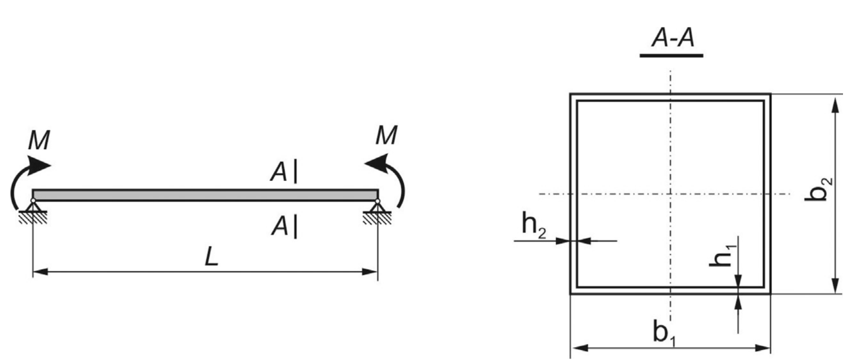

Fig. 3. Boundary condition, load and geometry of beam-column under consideration.