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Accurate buckling analysis of thin rectangular plates under locally distributed compressive edge stresses

Xinwei Wang *, Yu Wang, Luyao Ge

State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

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ABSTRACT

The buckling analysis of thin rectangular plates under locally distributed compressive edge stresses is a challenging problem if the point discrete methods are to be used. To obtain accurate buckling stress, one of the important factors is that the in-plane stress distributions within the plate prior to buckling should be accurate enough. Although it is possible to get analytical solutions for the in-plane stress distributions, but the expressions are very complicated since a stress-diffusion phenomenon exists. The differential quadrature method (DQM), being a point discrete method, has been successfully used in a variety of fields including the buckling analysis of thin rectangular plates under nonlinearly distributed edge compressions. However, it is rare to employ the DQM directly to solve problems of rectangular plates under locally distributed or point loads. To solve the challenging problem by using the DQM, novel formulations are presented in this paper. The locally distributed stress is first work-equivalently to point loads at all inner grid points on the loaded edge, then the normal stress boundary condition is numerically integrated before being discretized in terms of the differential quadrature. In this way accurate in-plane stress distributions can be obtained by the DQM without any difficulties. Buckling analysis of rectangular plates under either uniaxial or biaxial locally distributed compressive stresses is successfully performed. The accuracy of the DQ data is validated by comparing them with existing analytical solutions and finite element data. It is demonstrated that the compactness and computational efficiency of the DQM are retained. Accurate buckling loads are presented for rectangular plates with nine combinations of boundary conditions, various aspect ratios and load ratios. Some new results are also provided for references.

1. Introduction

The buckling problem of thin rectangular plates subjected to in-plane compressive and/or shear loading is important in the aircraft and civil industries [1]. Thin rectangular plates under locally distributed edge compressions, one of the most common load types in engineering practice [2], is a challenging problem since a stress-diffusion phenomenon exists. All three in-plane stress distributions vary with x as well as y and are very complicated. Therefore, obtaining accurate buckling load is not an easy task analytically. Very few analytical solutions are available for thin rectangular plates under nonlinearly distributed or locally distributed edge compressions [1,2].

Recently, Mijušković et al. [2] presented an accurate buckling analysis for thin rectangular plates under locally distributed compressive stresses. The accurate results are obtained by using the Ritz method together with the exact in-plane stress distributions. It is seen that analytical procedures for the exact stress and displacement function determinations in forms of series are very complex, thus commercial software such as Mathematica or Maple has to be resorted for symbolic computations.

Studies show that the differential quadrature method (DQM) can yield accurate buckling loads for rectangular plates under uniformly or non-uniformly distributed edge compressions [3,4]. For rectangular plates under cosine distributed edge compressions, the same accurate buckling stresses are obtained by the DQM as the ones with the exact in-plane stress distributions [2]. The solution accuracy is even better than the one of the finite element method (FEM) with fine meshes [2]. The DQM is simple and computationally efficient and has been successfully applied in a variety of fields [4,5]. Being a numerical method, the DQM can be used to obtain accurate solutions if the plate material is not isotropic [4].

Similar to the conventional point discrete methods such as the collocation method and finite difference method (FDM), however,

* Corresponding author.
E-mail address: wangx@nuaa.edu.cn (X. Wang).

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Nomenclature

DQM	differential quadrature method
FEM	finite element method
FDM	finite difference method
DQEM	differential quadrature element method
DQ	differential quadrature
a, b, h	length, width and thickness of a rectangular plate
E,μ	Young's modulus and Poisson's ratio of plate material
u, v	in-plane displacement components
w	transverse displacement
$\bar{\sigma}_{x0}$, $\bar{\sigma}_{y0}$, $\bar{\tau}_{xy0}$	applied edge normal and shear stresses
l_{x0}, l_x	starting x coordinate and the length where $\bar{\sigma}_{y0}$, $\bar{\tau}_{xy0}$ applied on $y = \pm b/2$
l_{y0}, l_y	starting y coordinate and the length where $\bar{\sigma}_{x0}$, $\bar{\tau}_{xy0}$ applied on $x = \pm a/2$
N	the total number of grid points in x and y directions
ξ_i, η_i	grid points in x and y directions
$r_y = l_y/b$, $r_{y0} = l_{y0}/b$	non-dimensional length and starting coordinate along edges $x = \pm a/2$
$r_x = l_x/a$, $r_{x0} = l_{x0}/a$	non-dimensional length and starting coordinate along edges $y = \pm b/2$
W_P	equivalent work done by the applied edge stress
ς_k, $H_k (k = 2, \ldots, N-1)$	abscissas and weights in Gauss quadrature
P_j	work equivalent point load
$l_j(\xi)$, $l_j(\eta)$	Lagrange interpolation function
$\delta(y - y_j)$	Dirac delta function
F_{xj}, F_{yj}	work equivalent stress applied on the edge grid point
A^x_{ij}, A^y_{lk}	weighting coefficients of the first order derivative w.r.t. x,y
B^x_{ij}, B^y_{lk}	weighting coefficients of the second order derivative w.r.t. x,y
C^x_{ij}, C^y_{lk}	weighting coefficients of the third order derivative w.r.t. x,y
D^x_{ij}, D^y_{lk}	weighting coefficients of the fourth order derivative w.r.t. x,y
D	flexural rigidity of the thin rectangular plate
N_x, N_y, N_{xy}	in-plane resultant force components
M_x, M_y	bending moments
MMWC	the method of modification of weighting coefficient
P_0	critical buckling load
λ	eigen-value
K	non-dimensional buckling coefficient
K_T	the non-dimensional buckling coefficient (point load)
C, S	clamped or simply supported boundary
f	ratio of applied edge normal stresses (σ_{y0}/σ_{x0})

the DQM has some difficulties in dealing with discontinuously or locally distributed loads. From the literature review, it is rare to use the DQM to solve problems in the area of structural mechanics involving discontinuously distributed loads and/or discontinuous boundary conditions. With uniformly distributing the concentrated load in a small area, the wavelet-based DQM [6] yields reasonably accurate deflection for a beam under a concentrated load. Han et al. [7] investigate the effect of different treatments of the concentrated load or locally distributed loads on the accuracy of DQ solutions. These treatments on the concentrated load or locally distributed loads are, however, only physically sound but mathematically not clear. The rate of convergence of the DQ solution is relatively low, thus only a little success is achieved in [6,7] to deal with the point and locally distributed loads. The problem involving discontinuously distributed loads and/or discontinuous boundary conditions is generally regarded as a challenge when point discrete methods such as the FDM and DQM are to be used for solutions.

According to the best of authors' knowledge, no one has used the conventional DQM to solve the buckling problem of thin rectangular plates under locally distributed loads thus far. Although the difficulties can be overcome by using the differential quadrature element method (DQEM) [4,8], however, the simplicity and computational efficiency will be lost. For thin rectangular plates under uniaxial or biaxial locally distributed stresses, the problems to be investigated herein, at least three or nine DQ elements are needed to model the entire plates. Therefore, the solution procedures will be complicated since assemblage procedures are needed. Using more DQ elements definitely reduces the computational efficiency. Therefore it is desirable to directly use the DQM to obtain accurate solutions.

Very recently, the buckling problem of rectangular plates under compressive point loads was successfully solved by the DQM [9]. The method to deal with the Dirac-delta function in [4] is used to treat the stress boundary conditions. A novel formulation is proposed to treat the point load. Accurate buckling loads are obtained by the DQM for thin rectangular plates under uniaxial and biaxial point loads. It is demonstrated that the advantages of the DQM, such as simple and computational efficient, are retained. However, the novel formulation presented in [9] is only limited to the cases when the point load is applied at the middle point of each edge. In other words, doubly symmetric condition can be used and the point load is always located at one grid point when DQM with odd number of grid points are used. In practice, the point load may be applied at locations other than the middle point of the edge.

In this paper, novel formulations are presented to deal with the locally distributed compressive edge stresses. The formulations are general enough and can be used for arbitrary locally distributed stresses. Detailed formulations and solution procedures by using the DQM are given. Convergence studies are performed. Some DQ results are verified by comparing them with existing analytical solutions and finite element data. Accurate buckling loads of thin rectangular plates with nine combinations of boundary conditions, various aspect ratios and load ratios are obtained by using the DQM together with the novel formulations. Some results are new and can be served as a reference for other researchers to develop new numerical methods. Finally some conclusions are drawn based on the results reported herein.

2. In-plane stress analysis by the DQM

Consider an isotropic thin rectangular plate with length a, width b and thickness h. The Young's modulus and Poisson's ratio are denoted by E and μ , respectively. Body force is not considered in present investigations.

In terms of displacement components u and v, the governing equations can be expressed by

$$\begin{cases} \dfrac{E}{1-\mu^2}\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{(1-\mu)}{2}\dfrac{\partial^2 u}{\partial y^2} + \dfrac{(1+\mu)}{2}\dfrac{\partial^2 v}{\partial x \partial y}\right) = 0 \\ \dfrac{E}{1-\mu^2}\left(\dfrac{\partial^2 v}{\partial y^2} + \dfrac{(1-\mu)}{2}\dfrac{\partial^2 v}{\partial x^2} + \dfrac{(1+\mu)}{2}\dfrac{\partial^2 u}{\partial x \partial y}\right) = 0 \end{cases} \tag{1}$$

The cases considered in this paper are rectangular plates under