Thin-Walled Structures 100 (2016) 124–133

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Postbuckling of pressure-loaded FGM doubly curved panels resting on elastic foundations in thermal environments

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A R T I C L E I N F O

Article history:
Received 31 October 2015
Received in revised form
20 November 2015
Accepted 20 November 2015
Available online 18 December 2015

Keywords:
Functionally graded materials
Doubly curved panel
Temperature-dependent properties
Postbuckling
Elastic foundation

A B S T R A C T

Modeling and analysis for the postbuckling of FGM doubly curved panels resting on elastic foundations and subjected to lateral pressure under heat conduction are presented. The initial deflections caused by lateral pressure and thermal bending stresses are both taken into account. The temperature-dependent material properties of functionally graded materials (FGMs) are assumed to be graded in the thickness direction based on Mori–Tanaka micromechanics model. The formulations are based on a higher order shear deformation theory with von Kármán strain–displacement relationships. The panel-foundation interaction and thermal effects are also included. The governing equations are first deduced to a boundary layer type that includes nonlinear prebuckling deformations and initial geometric imperfections of the panel. These equations are then solved by means of a singular perturbation technique along with a two-step perturbation approach. The effects of volume fraction index, temperature variation, the panel geometric parameters as well as foundation stiffness on the postbuckling behavior of FGM doubly curved panels are discussed in detail.

1. Introduction

The aerospace organizations are intensively involved in the development of advanced composite materials for analysis and design of aircrafts and space vehicles. In particular, many shell-type structures are designed to behave in a stable manner in the postbuckling range. Thus, a deep insight into the mathematical nature of their instability is essential for the prediction of the carrying capacity in the design process. Recently, a new class of composite materials known as functionally graded materials (FGMs) has drawn considerable attention. FGMs are heterogeneous composite materials usually made from a mixture of ceramics and metals. FGMs take advantage of heat and corrosion resistance of ceramics, and mechanical strength and toughness of metals. FGMs are characterized by a smooth and continuous change of the mechanical properties from one surface to another. A comprehensive survey of the relevant theoretical methodologies and modeling of FGM structures can be found in the review paper by Birman and Byrd [1] and a book by Shen [2]. Following these studies, extensive studies have been conducted on the bending, buckling and vibration of FGM shell panels with different boundary conditions [3–10].

Postbuckling is one of the major issues for FGM shell panels. Previous studies were mainly focused on the buckling and postbuckling of FGM cylindrical panels under axial compression. For example, Shen [11] presented the postbuckling analysis of FGM cylindrical panels subjected to axial compression in thermal environments based on a higher order shear deformation theory. This work was then extended to the cases of postbuckling of FGM cylindrical panels resting on elastic foundations subjected to axial compression in thermal environments and thermal postbuckling of FGM cylindrical panels resting on elastic foundations subjected to uniform and non-uniform temperature rise by Shen and Wang [12,13]. Yang et al. [14] studied thermal postbuckling behaviors of pre-axially-loaded FGM cylindrical panels by using a semi-analytical method based on the classical thin shell theory. Pradyumna and Bandyopadhyay [15] calculated the natural frequencies and buckling loads of FGM curved panels subjected to axial compression and a 1-D steady heat conduction by using finite element method (EFM) based on a higher order shear deformation theory. In the above studies [11–15], the material properties of FGMs are assumed to be temperature-dependent. On the other hand, Matsunaga [16] performed the free vibration and linear buckling of FGM cylindrical panels based on a 2-D higher order shear deformation theory. Duc and his co-authors [17–19] studied the buckling and postbuckling behaviors of FGM cylindrical panels without or resting on elastic foundations by using the Galerkin method based on the classical and higher order shear deformation theories, respectively. Liew and his co-authors [20,21] studied the buckling and postbuckling of FGM cylindrical panels subjected to

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http://dx.doi.org/10.1016/j.tws.2015.11.015

mechanical and thermal loads by using the element-free kp-Ritz method based on the first order shear deformation theory. In the above studies [16–21], however, the material properties of FGMs are assumed to be independent of temperature or to be fixed at room temperature $T=300$ K. In the aforementioned studies [11–21], the effective material properties of FGMs are usually assumed to obey from Voigt micromechanics model except for [12,13], in which Mori–Tanaka micromechanics model was adopted.

Solution of postbuckling behavior of curved panel subjected to lateral pressure is a more difficult task than the case of the same panel subjected to axial compression. This is because the curved panel will firstly bend under lateral pressure before the buckling occurs. It has been reported that the postbuckling behaviors of plates, shells and panels are different. Librescu and his co-authors [22,23] studied postbuckling of composite laminated and sandwich doubly curved panels under combined mechanical and thermal loadings. They concluded that the postbuckling equilibrium path of curved panels is no longer the bifurcation type when the panel subjected to lateral pressure. van Campen and his co-authors [24,25] studied postbuckling of orthotropic doubly curved panels subjected to lateral uniform pressure by using the adjacent equilibrium method based on the classical thin shell theory. They found that the postbuckling equilibrium path of curved panels is composed of two curves where the first curve is the bending curve and the second curve is the postbuckling curve and the transition point is defined as second bifurcation point. Duc and his co-authors [26,27] studied postbuckling of functionally graded ceramic-metal FGM doubly curved panels resting on elastic foundations subjected to thermomechanical loads by using the Galerkin method. They observed that the postbuckling equilibrium path is a snap-through type for both thin and shear deformable FGM spherical panels. Recently, Shen [28] studied postbuckling of FGM cylindrical panels resting on elastic foundations subjected to lateral pressure under heat conduction. The results reveal that the postbuckling equilibrium path of FGM cylindrical panels is stable and no snap-through phenomenon occurs. To the best of the authors' knowledge, there is no literature covering the nonlinear response of FGM doubly curved panels subjected to lateral pressure resting on elastic foundations under heat conduction.

In the present work, we focus our attention on the postbuckling of FGM doubly curved panels resting on elastic foundations and subjected to lateral pressure under heat conduction. Like in [28], the initial deflections caused by lateral pressure and thermal bending stresses are both taken into account. Unlike in [28] where the curvature effect is in one direction, the solution of the postbuckling behavior of a doubly curved FGM panel is more complicated due to the curvature effect in two directions. Mori–Tanaka micromechanics model instead of the commonly used Voigt micromechanics model is adopted to predict the effective material properties of FGMs. The formulations are based on a higher order shear deformation theory with von Kármán strain–displacement relationships. The panel-foundation interaction and thermal effects are also included. The material properties of FGMs are assumed to be graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents, and are assumed to be temperature-dependent. The governing equations are first deduced to a boundary layer type that includes nonlinear prebuckling deformations and initial geometric imperfections of the panels. These equations are then solved by means of a singular perturbation technique along with a two-step perturbation approach to determine the postbuckling equilibrium paths of doubly curved FGM panels.

2. Theoretical development

Consider an FGM doubly curved panel of rectangular planform resting on an elastic foundation, as shown in Fig. 1. A coordinate system (X, Y, and Z) is established for the panel in which X and Y are in the directions of the lines of curvature of the middle surface and Z is in the direction of the inward normal to the middle surface. The origin of the coordinate system is located at the corner of the panel in the middle plane. The displacements of the panel along the X, Y and Z directions are designated by $\bar{U}$, $\bar{V}$ and $\bar{W}$. $\bar{\Psi}_x$ and $\bar{\Psi}_y$ are the rotations of the normals to the middle surface with respect to the Y-and X-axes, respectively. The panel has length a, width b and constant thickness h. R_1 and R_2 are two radii of curvature to the middle surface. The panel is exposed to elevated temperature and is subjected to a transverse uniform pressure q. The foundation is assumed to be a compliant foundation of Pasternak-type, which means that no part of the panel lifts off the foundation in the large deflection region. The load–displacement relationship of the foundation is assumed to be $p_0 = \bar{K}_1\bar{W} - \bar{K}_2\nabla^2\bar{W}$, where p_0 is the force per unit area, $\bar{K}_1$ is the Winkler foundation stiffness and $\bar{K}_2$ is the shearing layer stiffness of the foundation, and ∇^2 is the Laplace operator in X and Y. The FGM panel is made from a mixture of ceramics and metals, the mixing ratio of which is varied continuously and smoothly in the thickness direction. We assume that the composition is varied from the outer to the inner surface, i.e. the outer surface ($Z=-h/2$) of the panel is ceramic-rich whereas the inner surface ($Z=h/2$) is metal-rich.

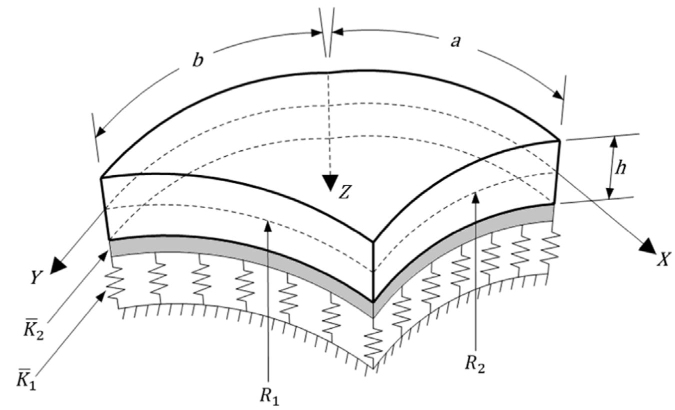

Fig. 1. Geometry and coordinate system of a double curved panel resting on a Pasternak elastic foundation.

Since ceramics and the metals used in FGMs do store different amounts of heat, this leads to a non-uniform distribution of temperature through the thickness for plate and shell structures, as reported in Praveen and Reddy [29]. In such a case, the temperature distribution along the thickness can be obtained by solving a 1-D steady-state heat transfer equation

$$-\frac{d}{dZ}\left[\kappa_f\frac{dT}{dZ}\right] = 0 \tag{1}$$

where κ_f is the thermal conductivity. This equation is solved by imposing boundary condition of $T=T_t$ at $Z=-h/2$ and $T=T_b$ at $Z=h/2$. The solution of this equation is

$$T(Z) = T_t - \frac{T_t - T_b}{\int_{-h/2}^{+h/2} dZ/\kappa_f}\int_{-h/2}^{Z}\frac{dZ}{\kappa_f} \tag{2}$$

Note that the temperature field is uniform when $T_t=T_b$.

The major difference between the conventional laminated composites and the FGMs lies in that the material properties of the latter, like the Young's modulus E_f, Poisson' ratio ν_f, thermal expansion coefficient α_f and thermal conductivity κ_f, are assumed to be both position (usually Z in the plate and shell structures) and temperature dependent. Several micromechanical models have