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A qualitative analytical investigation of geometrically nonlinear effects in wind turbine blade cross sections

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ABSTRACT

This paper analytically investigates the Brazier effect on asymmetric thin-walled sections subject to biaxial bending. In the latter case a torsional moment – in this paper referred to as *Brazier torsion* – is induced, which proved to be a vital part of the solution. By means of a generic cross section, that was inspired by a wind turbine blade, it is demonstrated that geometric nonlinear effects can induce an in-plane opening deformation in re-entrant corners that may decrease the fatigue life. The opening effect induces Mode-I stress intensity factors which exceed the threshold for fatigue crack growth at loads well below the load-carrying capacity of the beam.

The findings in this paper are twofold: Firstly, the investigated analysis procedure can be integrated into the design process of wind turbine blade cross sections. Secondly, the proposed approach serves as a basis for computationally efficient numerical analysis approaches of structures that comprise complex geometry and anisotropic material behaviour – such as wind turbine rotor blades.

1. Introduction

The investigation of tubes under large pure bending deformation was initiated by von Kármán [1] for curved circular tubes, and for rectangular hollow sections by Timoshenko [2], and was later modified for initially straight tubes by Brazier [3]. The well-known Brazier effect deals with the cross sectional capacity limit upon second-order in-plane deformations that lead to ovalisation of circular sections (Fig. 1(a)). The ovalisation is caused by in-plane deviation stresses as shown in Fig. 1(b). The Brazier effect has been studied extensively by many researchers, such as Clark and Reissner [4] and Reissner [5] to name only a few. Recently Guarracino [6] investigated the formation of axial wrinkles at the compressed region where Corona and Rodrigues [7] investigated bifurcation buckling of orthotropic circular tubes. This shows that research on the Brazier effect focussed earlier on the second-order capacity limits of tubular cross sections, but more recently on the transition of the Brazier effect into local stability limits (i.e. local buckling). Moreover, Guarracino et al. [8–10] experimentally, numerically and analytically investigated local effects imposed by collars and supports as they e.g. appear in pipelines. They pointed out that counterintuitive implications of the Brazier effect can cause significant deviations of axial tensile and compressive strains obtained from simple bending theory.

Although the Brazier effect is present in all cross sections it becomes mostly apparent in thin-walled hollow sections. A typical representative of such a thin-walled multi-cellular cross section is that of a wind turbine blade as depicted in Fig. 1(c).

Wind turbine blades are cantilever glass fibre reinforced polymer composite beams that can undergo considerable bending deformations of up to 20% of their span. In the design of wind turbine blades the classic Brazier limit is usually not relevant. Kühlmeier [11] showed that the local buckling limit of wind turbine blades is usually lower than the Brazier limit moment. Nevertheless, Damkilde and Lund [12] showed that the Brazier effect induces transverse stresses in the cap of the main carrying box girder of wind turbine blades. Due to the distinct orthotropic material behaviour of caps, it was shown that these transverse stresses can reach the transverse tensile strength of the material at load levels below the Brazier limit. This finding was significant as the structural impact of the Brazier effect can no longer be neglected in the blade design process.

Recent experimental and numerical investigations of fracture mode deformations near the trailing edge of a wind turbine blade [13] showed that in-plane deformations could be attributed to the Brazier effect. More importantly it could be seen that the trailing edge 'opened' for certain loading directions and load levels far below the capacity limit of the blade, all of which make the trailing edge susceptible to fatigue damage.

It was Cecchini and Weaver [14] who initially shed light on geometric nonlinear effects affecting the trailing edge. They provided an analytical approach for the Brazier effect in a simplified symmetric airfoil that is subjected to pure flapwise bending. Following the

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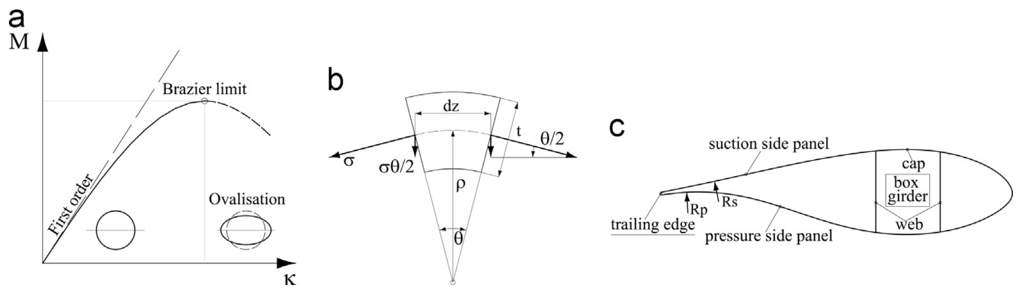

Fig. 1. (a) Bending moment curvature responses for first-order (dashed line) and second-order (solid line) cross-section capacities due to ovalisation; (b) infinitesimal element subject to bending stress σ and second order in-plane deviation stress responsible for Brazier effect; (c) sketch of a typical wind turbine rotor blade cross section; the aerodynamic shell and the box girder form a tri-cellular cross section; pressure side panel and suction side panel are joined at a re-entrant corner which is referred to as trailing edge. Both panels feature a curvature whose local radii are denoted as R_p and R_s, both of which have a significant influence on the aerodynamic performance of the airfoil.

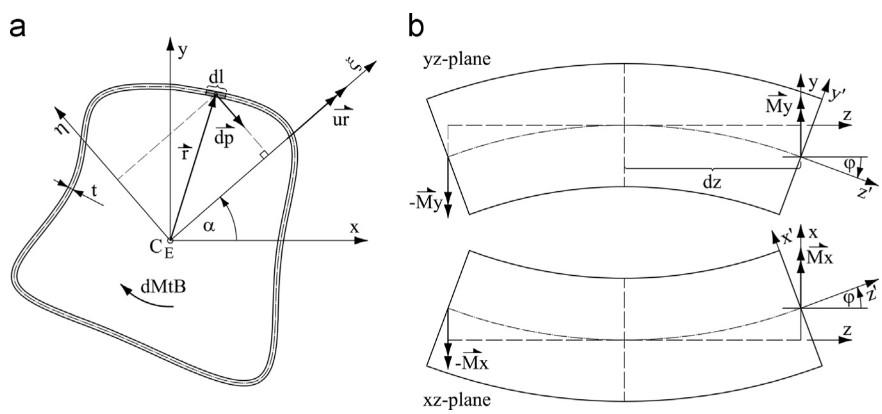

Fig. 2. (a) Arbitrary thin-walled tubular cross section subjected to biaxial bending denoted by rotation vector $\overrightarrow{ur}$. Principal cross section coordinate system x, y and the bending coordinate system aligned with bending axis ξ, η; (b) deformed main beam element (Euler–Bernoulli hypothesis) subjected to biaxial bending $\overrightarrow{M_x}$, $\overrightarrow{M_y}$ depicted in two orthogonal views with the coordinate system of the deformed cross section x', y'.

concept of Brazier, they reduced the 3D problem to a 2D problem which can be solved both analytically and numerically on a cross-sectional level. Cecchini and Weaver subsequently proposed a finite-element-based approach in which fully non-linear deformations can be efficiently obtained from a thin cross sectional slice. They eventually showed – by means of a NACA0012 profile [15] – that flapwise bending (bending around the minor principal axis) leads to a closing of the trailing edge. This behaviour can be intuitively explained by the symmetrically distributed Brazier pressure that consequently squeezes the trailing edge panels together.

However, the trailing edge deformation behaviour becomes less intuitive for asymmetric, curved, thin-walled sections subject to bending about other than the principal axes. The analysis of such a general case is not straightforward owing to a torsional moment that is induced into the deformed cross section which must be considered in order to satisfy the equilibrium conditions. It will be shown in this paper that certain asymmetric, curved thin-walled sections exhibit a counterintuitive opening effect of the trailing edge under certain bending directions which is consistent with both experimental and numerical analyses.

The aim of this paper is to analytically investigate the source of this effect in order to aid the future development of structurally improved airfoil geometries. Owing to the complexity of wind turbine blades with regard to geometry and anisotropic material behaviour this effect will be demonstrated on a simplified example. Moreover, it must be stressed that nonlinear numerical fracture analyses of 3D wind turbine rotor blade models are computationally expensive. This paper follows the modelling approach of [14] and aims to facilitate a more efficient numerical analysis approach of asymmetric airfoil slices for bi-axial bending directions.

2. Methods

2.1. Brazier torsion

The Brazier pressure acting at a point on the centre line of the isotropic, linear elastic, thin-walled section shown in Fig. 1(b) can be written with $\theta = \mathrm{d}z/\rho = \kappa\, \mathrm{d}z$ as

$$\mathrm{d}p = \sigma t \theta = \kappa^2 E t r\, \mathrm{d}z \tag{1}$$

where θ represents the angle of bending rotation, ρ denotes the bending radius, κ represents the St. Venant bending curvature, E denotes the elastic modulus, t denotes the wall thickness, r represents the distance from that point perpendicular to the bending axis and dz represents the thickness of an infinitesimal cross sectional slice of the main beam. The Brazier pressure has the unit *force per circumferential wall length* dl. Clearly, the Brazier pressure vector always points perpendicularly towards the bending axis (i.e. axis of curvature). With the position vector $\vec{r} = \{x, y\}^T$ and using the transformation matrix

$$T = \begin{bmatrix} \cos\,\alpha & -\sin\,\alpha \\ \sin\,\alpha & \cos\,\alpha \end{bmatrix} \tag{2}$$

the pressure Brazier vector can be written as

$$\mathrm{d}\vec{p} = -\kappa^2 E t\, \mathrm{d}z \begin{Bmatrix} \sin\,\alpha^2 x + \sin\,\alpha\,\cos\,\alpha y \\ \sin\,\alpha\,\cos\,\alpha x + \cos\,\alpha^2 y \end{Bmatrix} \tag{3}$$

where x and y are the Cartesian coordinates of an arbitrary point on the wall with its origin in the elastic centre C_E and α represents the angle between the bending axis (i.e. axis of cross section rotation) and the positive x-axis as shown in Fig. 2(a).