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Theoretical investigation of the strength and stability of special pseudospherical shells under external pressure

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ABSTRACT

The paper is devoted to a new shape of a shell of revolution with negative Gaussian curvature. The main part of the meridian of the shell is a plane curve of the Huygens tractrix. Geometrical properties of the middle surface of the shell of revolution are presented. The membrane state of stress for a family of shells with constant capacity and constant mass under uniform external pressure is analysed. The critical pressure, buckling modes and equilibrium paths for the family of shells are calculated with the use of the FEM (the ANSYS system). Results of the analytical and numerical investigations are presented in tables and figures. A stable post-critical behaviour of presented shells is pointed out which is not typical for most shell structures.

1. Introduction

The post-buckling behaviour of elastic structures is a significant design feature. The knowledge of the equilibrium paths is necessary for the design engineering with respect to operational safety of machinery. Equilibrium paths of classical shells of revolution with positive Gaussian curvature are unstable. Objective basic research for these problems was presented by Hutchinson and Koiter [1], Budiansky [2], Simitses [3] and Riks [4]. Numerical methods in buckling analysis of shells were described by Bushnell [5]. Reviews of buckling problems of thin shells are presented by Teng [6], Krivoshapko [7,8], Jasion and Magnucki [9], and Pan and Cui [10]. Asymptotic methods in the buckling analysis of elastic shells were elaborated by Tovstik and Smirnov [11]. The post-buckling behaviour of elastic shells of revolution was studied by Grigolûk and Lopanitsyn [12], Teng and Hong [13] and Kere and Lyly [14]. Problems of stabilisation of equilibrium paths for elastic structures were studied by Bochenek and Kruźelecki [15], Bochenek and Foryś [16], Bielski and Bochenek [17], Król et al. [18], Kruźelecki and Trybuła [19–21]. The numerical analysis of equilibrium paths of shells of revolution with positive and negative Gaussian curvature was presented by Jasion [22,23] and Jasion and Magnucki [24].

The subject of the present paper is cylindrical–pseudospherical and tori-pseudospherical shells of revolution. The meridians of these structures are the plane curves composed of a line segment and the Huygens tractrix or a circular arc and the Huygens tractrix. The goal of the present investigation is to determine the stress distribution in such shells, to analyse the influence of the geometrical parameters on the buckling load and buckling shape and to determine the character of the post-buckling behaviour of proposed shells. The last goal is of the most importance since, as it was shown in [22,23], for shells with negative Gaussian curvature this behaviour may be stable.

2. Special pseudospherical shells of revolution

2.1. Geometry of the middle surface of the pseudospherical shell

The surface called a pseudosphere can be formed by rotation of the Huygens tractrix around the x-axis (Fig. 1). The Huygens tractrix curve [25] is defined as follows:

$$\tilde{x} = \pm\left[\operatorname{arccosh}\left(\frac{1}{\tilde{r}}\right) - \sqrt{1-\tilde{r}^2}\right], \tag{1}$$

where $\tilde{x} = x/a$, $\tilde{r} = r/a$ – dimensionless coordinates, a – positive constant (the size). Parameterisation in the Cartesian coordinates

$$\tilde{x} = \ln\left(\tan\frac{\alpha}{2}\right) + \cos\alpha, \quad \tilde{r} = \sin\alpha, \quad 0 \le \alpha \le \pi. \tag{2}$$

The principal curvature radii are as follows:

$$\tilde{R}_1 = \frac{1}{\tan\alpha}, \quad \tilde{R}_2 = -\tan\alpha \tag{3}$$

where $\tilde{R}_1 = R_1/a$ – the dimensionless principal radius of the meridian, $\tilde{R}_2 = R_2/a$ – the dimensionless principal radius of the

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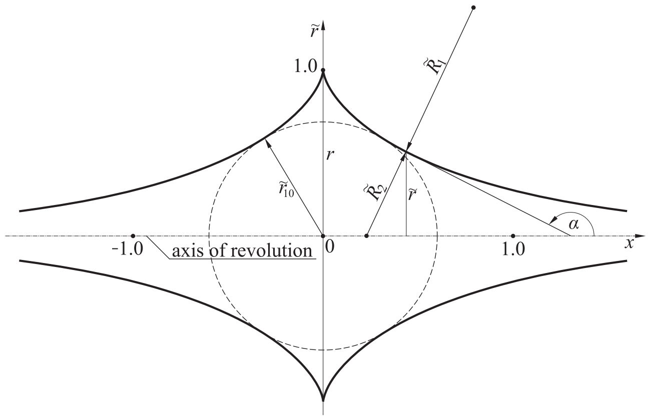

Fig. 1. The pseudosphere.

parallel circle. The Gaussian curvature $K = 1/(R_1 R_2) = -1/a^2$ is constant and negative for the whole pseudosphere.

2.2. Cylindrical–pseudospherical shell

The meridian of the cylindrical–pseudospherical shell is composed of a line segment and the Huygens tractrix (Fig. 2). The capacity of the shell (Fig. 2) is given by the following formula:

$$V_0 = 2\pi a^3 \left(\tilde{x}_1 \tilde{r}_1^2 + \int_{\tilde{x}_1}^{\tilde{x}_2} \tilde{r}^2 \, d\tilde{x} \right) = 2\pi a^3 \left(\tilde{x}_1 \tilde{r}_1^2 + \int_{\alpha_2}^{\alpha_1} \cos^2\alpha \sin\alpha \, d\alpha \right). \tag{4}$$

After integration the size a can be written as

$$a = \left[\frac{3V_0}{2\pi\left(3\tilde{x}_1 \tilde{r}_1^2 + \cos^3\alpha_1 - \cos^3\alpha_2\right)} \right]^{1/3}. \tag{5}$$

For the pseudospherical shell ($\tilde{x}_1 = 0$, $\alpha_1 = \pi/2$, $\alpha_2 = \pi$) the capacity $V_0^{(ps)} = (2/3)\pi a^3$. The mass of the shell (Fig. 2)

$$m_s = A_s t_s \rho_s, \tag{6}$$

where the lateral area

$$A_s = 4\pi a^2 \left(\tilde{x}_1 \tilde{r}_1 + \frac{1}{2}\tilde{r}_2^2 + \int_{\alpha_2}^{\alpha_1} \tilde{r}\tilde{R}_1 \, d\alpha \right). \tag{7}$$

Taking into account Eqs. (2) and (7) the thickness of the shell, based on Eq. (6), can be expressed as

$$t_s = \frac{m_s}{2\pi[2(1+\tilde{x}_1)\tilde{r}_1 - (2-\tilde{r}_2)\tilde{r}_2]a^2 \rho_s}, \tag{8}$$

where ρ_s – mass density of the shell material. For the pseudospherical shell ($\tilde{x}_1 = 0$, $\alpha_1 = \pi/2$, $\alpha_2 = \pi$) the lateral area $A_s^{(ps)} = 4\pi a^2$.

The assumption of the values of the capacity V_s (m^3) and the mass m_s (kg) for the shell enables us to calculate the size a and the thickness t_s from (5) and (8), respectively.

2.3. Tori-pseudospherical shell

The meridian of the tori-pseudospherical shell is composed of a circular arc and the Huygens tractrix (Fig. 3). The toroidal part of the shell is shown in Fig. 4. The dimensionless radius of the torus

$$\tilde{r}_0 = \frac{\tilde{x}_1}{\sin\alpha_1} = \frac{\tilde{x}_1}{\tilde{r}_1}, \tag{9}$$

and the dimensionless coordinate

$$\tilde{r}(\varphi) = \tilde{r}_1 + \tilde{r}_0(\cos\alpha_1 + \cos\varphi), \quad 0 \le \varphi \le \pi - \alpha_1. \tag{10}$$

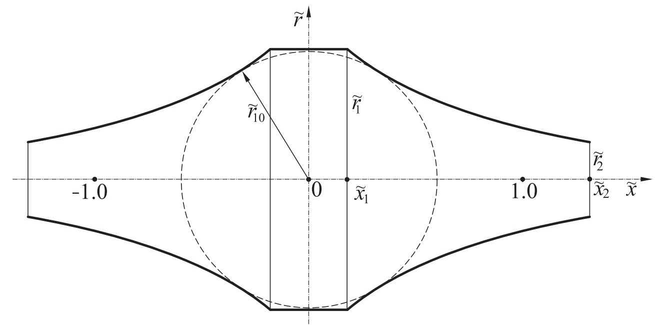

Fig. 2. The cylindrical–pseudospherical shell.

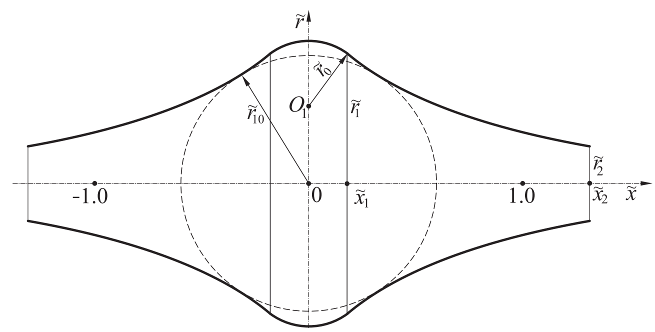

Fig. 3. The tori-pseudospherical shell.

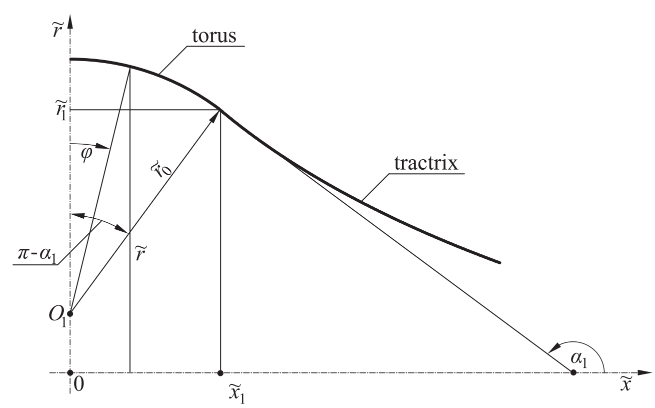

Fig. 4. Toroidal part of the shell of revolution.

The capacity of the shell (Fig. 3)

$$V_0 = 2\pi a^3 \left\{ \int_0^{\tilde{x}_1} [\tilde{r}(\varphi)]^2 \, d\tilde{x} + \int_{\tilde{x}_1}^{\tilde{x}_2} [\tilde{r}(\alpha)]^2 \, d\tilde{x} \right\}. \tag{11}$$

After integration the size a can be written as

$$a = \left[\frac{3V_0}{2\pi(3f_{vt} + \cos^3\alpha_1 - \cos^3\alpha_2)} \right]^{1/3}, \tag{12}$$

where

$$f_{vt} = \left[\tilde{r}_1^2 + \tilde{x}_1 \cos\alpha_1 + \frac{1}{3}\left(3 - \tilde{r}_1^2\right)\tilde{r}_0^2 \right] \tilde{x}_1 + (\pi - \alpha_1)\left(\tilde{x}_1 + \tilde{r}_0^2 \cos\alpha_1\right)\tilde{r}_0. \tag{13}$$

The mass of the shell (Fig. 3) is given by Eq. (6). The lateral area A_s is

$$A_s = 4\pi a^2 \left(\frac{1}{2}\tilde{r}_2^2 + \tilde{r}_0 \int_0^{\pi-\alpha_1} \tilde{r}(\varphi) \, d\varphi + \int_{\alpha_2}^{\alpha_1} \tilde{r}\tilde{R}_1 \, d\alpha \right). \tag{14}$$