Thin-Walled Structures 93 (2015) 112–121

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Selected problems concerning determination of the buckling load of channel section beams and columns

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ARTICLE INFO

Article history:
Received 3 February 2015
Received in revised form
10 March 2015
Accepted 10 March 2015
Available online 4 April 2015

Keywords:
Thin-walled structure
Buckling load
Experimental investigations
Composite

ABSTRACT

The issue of buckling load determination in composite channel section beams subjected to pure bending and channel section columns subjected to uniform compression is considered. Some selected problems with determination of buckling load on the basis of the collected and processed experimental data are discussed. The data necessary to determine buckling load (applied load and the corresponding displacement, strains at chosen points of beam-columns and displacements in three perpendicular directions of all visible points of the considered beam-columns) were collected with a strain gauge system, an Aramis® 3D optical system and a universal testing machine. Buckling load was determined by means of the following well-known methods: the mean strain method, the method of straight-lines intersection on the graph of load vs. mean strains, the curve inflection point method, the load-square of deflection curve method and Koiter's method. All results were obtained during the experimental investigations and the numerical FEM analysis of the channel section profile made of a GFRP laminate with the symmetrical eight-layer arrangement $[45/\text{-}45/90/0]_s$. The profiles under consideration were subjected to compression or pure bending (four-point bending test). The rules for each methods of buckling load determination are proposed on the basis of the obtained results.

1. Introduction

Thin-walled structures made of traditional (steel, aluminium) or composite materials are used in different branches of industry. Thin plates or thin-walled structures are used in sports and automotive industry, aerospace and civil engineering. Snowboards, skis or poles, as well as all kinds of crane girders [29], structural components of automobiles [1] (car body sheathing and all longitudinal members), aircraft fuselages [26] and wings [3], supporting structures of walls and roofs of large halls and warehouses [4] can be mentioned as examples of such structural elements.

It is obvious that buckling can lead thin-walled structures to collapse. However, it should be noted that plate structures can work in the postbuckling range as well, although they are characterised by lower stiffness then. Therefore, determination of buckling load is essential from the design and engineering point of view.

Scientists have been developing methods of buckling load determination for more than a hundred years. Here, the precursors like Euler [5], Volmir [36] and Timoshenko [33] should be mentioned. The major development of research on stability of thin-walled isotropic structures took place in the 1970s and the 1980s. The exemplary papers dealing with local buckling of thin-walled structures are those written by Davis and Hancock [7], Graves-Smith [11] or Mulligan and Pekoz [25]. The buckling of open section beams was the subject of the papers written by Magnucki [14,24], Ovesy [27], Macdonald [21,23]. In the worldwide literature, studies into nonlinear problems of stability of thin-walled structures made of orthotropic materials can be found easily. The oldest work on this subject was published almost 80 years ago. A broader literature overview of publications devoted to buckling and postbuckling behaviour of isotropic and orthotropic thin-walled structures can be found, for example, in Kubiak [19] or Kołakowski and Kowal-Michalska [17].

For real structures which have different types of imperfections (geometrical initial deflections, inhomogeneity of structures, residual stresses, etc.), experimental investigations ought to be performed. In such a case, special methods for buckling load determination should be employed. In the world literature, the mean strain method [13,30,35], the method of straight-lines intersection in the graph of mean strains [13,30,35], the load vs. square of strain difference $P\text{–}(\varepsilon_1\text{–}\varepsilon_2)^2$ method [13,30,35], the load vs. strain difference $P\text{–}(\varepsilon_1\text{–}\varepsilon_2)$ curve inflection point method [13], Tereszkowski's method [32] and Koiter's method [12,18] can be found. The

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http://dx.doi.org/10.1016/j.tws.2015.03.009
0263-8231/

above-mentioned methods of buckling load determination have been presented also by Rhodes and Zaras [28]. Debski et al. [8–10] have used these methods as well. Another very popular method for determination of buckling load is Southwell's method [2,31,34,37]. However, this method is rather used for compressed columns.

It should be noted that the authors of this paper cannot find the exact (step by step) explanation of application of the above-mentioned methods for buckling load determination. In the literature, there is a lack of explanations of the following exemplary dilemmas concerning the use of the mentioned methods:

- What kind of function should be used for approximation of the experimental results?
- How to find the inflection point of the function when its second derivative has more than one root?
- What range of experimentally gathered data should be taken into account during the processing?
- What is the most reliable method of buckling load determination?
- How to choose the method of buckling load determination for the given type of structures and load?

These questions and difficulties in finding the answers have been the reason why the authors of this paper have decided to investigate buckling load determination on the basis of the experimental data.

The experimental investigations on the buckling and postbuckling behaviour of thin-walled GFRP laminate channel section beams and columns with different layer arrangements were conducted in the Department of Strength of Materials at the Lodz University of Technology [6,20]. Both compression and four-point bending tests were performed. A channel section profile with the symmetrical layer arrangement $[45/\text{-}45/90/0]_s$ was chosen for accurate investigations of buckling load determination and a possible way of application of the employed methods for buckling load determination is presented. The aim of this study is to discuss the problems that arise while using the known methods of determination of buckling load. These investigations are also an attempt to answer the questions of how important it is to proceed according to a particular fixed pattern within a given method and which methods are the most suitable and the easiest to use in the cases under consideration.

2. Experimental investigations

A channel section profile made of an eight-layer laminate of the glass fibre reinforced polymer prepreg with the cross-section dimensions presented in Fig. 1 was considered. Different layer arrangements were tested [6], but the discussion of the methods of buckling load determination was conducted for one lay-up case, i.e., $[45/\text{-}45/90/0]_s$. The length of the compressed column was $L=250$ mm, and for the beam subject to pure bending, the part of the beam under consideration (profile between grips) was 275 mm long.

The investigated composite columns and beams were subjected to compression or pure bending, respectively. In the case of pure bending, load was applied in a four-point bending test. A scheme of the performed compression and bending tests is presented in Fig. 2.

All tests were performed on an INSTRON testing machine modernized by Zwick/Roell and equipped with specially designed grips. The values of the loading force applied to the system and the displacement in points where the load was applied were obtained directly from the machine sensors. In addition, strain gauges were stuck on the profiles. As regards the bending test, the strain gauges were located in the geometrical centre of the web on both sides of the wall. For specimens subjected to compression, the strain gauges were located in the centre of the first halfwave. This position was based on the numerical calculations. Additionally, a stereographic optical system [15,22] (Aramis®) for non-contact displacement and strain measurements was employed.

3. Methods applied for buckling load determination

On the basis of the collected data, buckling loads were determined according to the following five methods:

- mean strain method [13,30,35]—denoted as M-1;
- method of straight-lines intersection in the graph of mean strains [13,30,35]—denoted as M-2;
- load vs. square of strain difference $P\text{–}(\varepsilon_1\text{–}\varepsilon_2)^2$ method [13,30,35]—denoted as M-3;
- Koiter's method [12,18]—denoted as M-4;
- load vs. strain difference $P\text{–}(\varepsilon_1\text{–}\varepsilon_2)$ curve inflection point method [13]—denoted as M-5.

In the vertical tangent method (M-1), a diagram presenting the applied load as a function of the mean strain is used (Fig. 3). The mean strain is obtained as an average of the data registered with two strain gauges mounted along the loading direction on both sides of a wall of the specimen near the expected highest deflection area. The graph is plotted with a spline curve approximating the points measured. The value of buckling load is obtained from the Y-coordinate of the contact point of the plotted curve and the vertical line tangent to the curve. This method is very clear and explicit when compared to other methods. There is only one point of tangency which is not dependent on the points at the beginning and the end of measurement. This method could be merely invalid in the case there would be too few measurements made and no explicit bent of the curve would be drawn.

For the method denoted as M-2, a graph is plotted in the same way as in the M-1 method (Fig. 4). It is essential to notice that the curve presenting a relation of load vs. mean strain can be divided into two parts—the lower part for the prebuckling state and the higher one for the postbuckling state. In the method of straight

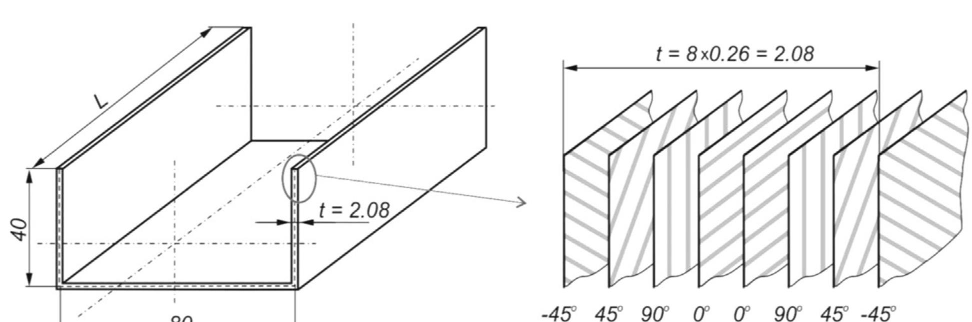

Fig. 1. Cross-section dimensions of the considered profile.