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Simulation of buckling process of cylindrical metal silos with flat sheets containing bulk solids

M. Wójcik, J. Tejchman *

Gdańsk University of Technology, Narutowicza 11/12, 80-233 Gdańsk-Wrzeszcz, Poland

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ABSTRACT

The paper presents 3D results on stability of thin-walled cylindrical metal silos made from isotropic rolled plates containing bulk solids. The behaviour of bulk solids was described with a hypoplastic constitutive model. Non-linear FE analyses with both geometric and material non-linearity were performed with a perfect and an imperfect silo shell wherein 3 different initial geometric imperfections were taken into account. The influence of a stored bulk solid (dry medium-dense cohesionless sand) during filling on the buckling strength of silos was compared with the strength of an empty silo and with the experimental results available in the literature. Our numerical results indicate a clear strengthening effect of the stored solid on the silo buckling strength as in the experiments, depending upon the wall thickness, wall loading way and imperfection type and amplitude.

1. Introduction

Thin-walled metal cylindrical shells made from isotropic rolled plates welded along the perimeter are frequently used in the silo industry. They are vulnerable to buckling caused by the wall friction force caused by the interaction between the silo fill and the silo wall, particularly during eccentric discharge, which is usually difficult to avoid with regard to a non-homogeneous character of bulk solids. The buckling strength of shells depends on many different factors such as: form and amplitude of initial geometric imperfections, loading and material imperfections, type of joints, boundary conditions at ends, flow pattern, level of internal pressurization and stiffness of the stored bulk solid [1,2]. The buckling strength of silo shells containing bulk solids at rest may be significantly enhanced when compared to empty silos due to both the internal pressure and lateral support produced by the silo fill [3–11]. On one hand, the internal pressure in bulk solids acting on the silo wall straightens wall imperfections and increases the buckling strength. On the other hand, the silo walls are supported by the fill which restraints them against buckling. The experiments with cylindrical model silos with isotropic flat walls containing bulk solids show evidently a considerable increase of the buckling strength of full silos against empty ones due to both the internal pressure [12,13] and the bedding effect [3,8,10,14,15]. The strength's increase exceeds even 100%. In rectangular silos, the buckling strength has not to be enhanced because the plates can buckle also in the outward direction. A positive bedding effect of a silo fill on the stability of cylindrical silos was mainly observed during silo filling. For silo emptying, this effect strongly depends upon a flow pattern and was not definitely determined in experiments [8,10]. In the case of mass flow, strong dynamic effects may occur and the silo can buckle at lower loads. The positive bedding effect is not taken into account in silo standards.

The aim of experimental and numerical research works performed at Gdańsk University of Technology is to determine in numbers the effect of the presence of the bulk solids on the buckling strength of cylindrical thin-walled metal silos (with flat and corrugated sheets) during both filling and emptying, by taking into account the stiffness and initial density of the bulk solid and the pressure level. In the first step, the behaviour of a cylindrical metal silo composed of flat sheets containing dry cohesionless medium-dense sand was investigated during filling. In this paper, some three-dimensional non-linear dynamic stability finite element analyses were carried out to determine the influence of the sand stiffness on the buckling strength with the commercial programme ABAQUS [16]. A hypoplastic constitutive model for sand was implemented and used to describe the behaviour of sand [17–19]. Since the silo walls were assumed to be smooth, the enhancement of a constitutive model by micro-polar, non-local or gradient terms [20] was not needed during silo filling due to a small shear strain level in sand at smooth walls. The constitutive model was enhanced by an intergranular strain concept which enables to capture the sand elasticity [21]. Non-linear analyses with the geometric and material non-linearity were carried out

* Corresponding author.
E-mail addresses: mwojcik@pg.gda.pl (M. Wójcik), tejchmk@pg.gda.pl (J. Tejchman).

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with a perfect and an imperfect silo shell [22,23]. Three different initial geometric imperfections were assumed. The FE results were compared with corresponding experiments performed at University of Karlsruhe [8,10]. The buckling deformation of silo walls was induced either by a vertical displacement prescribed to its top edge (as in model tests) or by a silo filling process (as in real silos).

The innovative point is that the stiffness of the bulk solid was taken into account in stability studies of a silo shell wherein its behaviour was described with a realistic constitutive model. Such calculations have not been performed to our knowledge yet.

2. Literature review

Several theoretical stability analyses of cylindrical silos with isotropic walls with consideration of a bedding effect of the silo fill confirm also a strengthening effect of bulk solids [6,8,10,24–27]. In these analyses, two factors were crucial for the determination of the shell stability: type of initial wall imperfections inducing buckles and the magnitude of the bedding modulus of the silo fill (i.e. modulus of sub-grade reaction) expressed by the ratio between the solid pressure and the wall deflection. The presence of small geometric imperfections in the wall greatly weakens its resistance to buckling [28]. The imperfections have two main contributing reasons: the plate rolling process producing plates with curled edges and the contraction of the circumferential weld during cooling which induces an inward radial displacement [25,29]. The resulting imperfections are generally locally axisymmetric [25]. Different forms and amplitudes of initial imperfections were assumed to find the maximum reduction of the buckling strength in shells with isotropic walls [6,24,25,29–39]. One assumed the imperfections in the form of regular harmonic pattern in the vertical and horizontal direction, in the form of eigen-modes of the perfect structure [38] and in the form of an irregular pattern on the basis of measurements [8,10,40–42]. The imperfections were fully axisymmetric, locally axisymmetric or completely local [35,36,41]. The fully axisymmetric imperfections in the form of a sinusoidal function resulted in the strongest reduction of the buckling load [25,28,33,34]. However, an axisymmetric local imperfection in a geometric form proposed by Berry and Rotter [25] and Rotter [4,43] (on the basis of measurements of a weld shrinkage depression profile) turned out to be the most realistic choice for practical design calculations in relation to full-scale structural measurements. Such locally axisymmetric imperfection with an angular extent of 40° can reduce the strength to the same extent as a fully axisymmetric imperfection [4]. The sufficient depth h of the imperfection was assumed to equal to $(1-2)\times t$, where t – the wall thickness.

In turn, the bedding effect was modelled in silos with isotropic walls mainly by means of simple linear springs [44] which described the silo fill foundation using the Winkler model. Knödel [8] improved the Winkler model by introducing a trilinear law which simulated also the ideal plastic behaviour of solids and prevented the existence of tensile stresses between the shell wall and the bulk solid. The non-linear springs were also used [6]. In all these theoretical models, the horizontal springs (radial-orientated) were prescribed along the silo wall and their stiffness was roughly estimated with the aid of tests, analytical formulas or FE-simulations. Thus, only solid resistance against the normal stress was taken into account [45]. To describe the shear resistance of bulk solids, Ummenhofer [10] proposed transverse springs between horizontal ones according to the Pasternak foundation model but no calculations were performed. In the calculations of the silo stability, the effects of the stress level, deformation direction and change of the solid density on the springs' stiffness were neglected [45].

Stability analyses for imperfect engineering structures are usually carried out by means of a linear buckling analysis and non-linear static studies based on the equilibrium path between the load and displacement. In order to capture the unstable post-buckling behaviour, the arc-length method by Riks is used that is the most fundamental procedure for a stable analysis process under the global load control. It works well in snap-through problems, where the equilibrium path in a load–displacement space is smooth and does not branch. However, the solution is sometimes impossible to be achieved due to the convergence loss caused by a localized instability (e.g. surface wrinkling, local buckling or material instability), stiffness lack at the limit point and a post-critical buckling event is a dynamic phenomenon. Many loops with several limit points may occur on the equilibrium path plots which contribute to convergence difficulties. In order to overcome these problems and to reach a solution, dynamic analyses may be applied [46–53], where the time history of a structure response is traced during growing load and the presence of inertial forces and damping improves the solution convergence. Alternatively, artificial damping can be used in static analyses. The advantage of dynamic analyses as compared to static analysis concerns the ensured convergence of the load–displacement path equilibrium in a post-peak regime, when a local transfer of strain energy from one part of the structure to its neighbouring parts occurs (local instability) and a global solution method may not work. The application of static stability analyses is certainly more realistic in a pre-peak regime of buckling, however in a post-peak regime this assumption is not unique, since the stability loss connected usually with structural stability jumping modes has a dynamic character. A dynamic stability analysis is also physically justified in silo shells under compression since buckles have a dynamic character [8,10]. In addition, during silo emptying strong dynamic effects can happen [54–56].

3. Experimental results

The comprehensive silo model experiments (medium-scale) were carried out at Karlsruhe University [6–11]. The height of the steel cylindrical model silo was 5.17 m and the diameter was 1.25 m (Fig. 1). The silo consisted of 4 rings (each 1 m high) and a mass flow hopper. The thickness of the 3 upper rings was 2 mm. The lower fourth ring was thinner, i.e. $t=0.625$ mm, $t=0.75$ mm or $t=1$ mm in order to induce the buckling failure in a model silo (t – wall thickness). The yield stress was 180 MPa for the rings of $t=0.625$ mm and $t=1.0$ mm and 370 MPa for the ring of $t=0.75$ mm. The experiments were performed with an empty silo and with a silo containing the so-called 'Karlsruhe' sand' with the mean grain diameter of $d_{50}=0.5$ mm (dry cohesionless sand). The initial wall imperfections for the lower ring thickness of $t=0.75$ mm were measured to be smaller than 1.13 mm ($1.5\times t$) in a vertical and horizontal direction (Fig. 2). Thus, they were smaller than the 'normal' fabrication tolerance w_{max} for usual metal constructions according to Eurocode 3 [57] (with the quality parameter $Q=16$)

$$w=\frac{t}{Q}\sqrt{\frac{r}{t}}=1.35\text{ mm}, \tag{1}$$

where $t=0.75$ mm is the wall thickness and $r=0.625$ m is the silo radius. In turn, the silo ring thickness of $t=0.625$ mm and $t=1.0$ mm was fabricated with a worst quality class (with the maximum imperfection amplitude equal to $w=3.6$ mm). Since the wall friction force produced by the silo fill was not enough to contribute to silo buckling in the lowest ring, an additional vertical load was imposed on the silo top edge [8,10].

The experimental data showed that

– the experimental load buckling factor $\alpha=\sigma_u/\sigma_b$ was 0.21–0.32 for the empty silo and 0.38–0.68 for the silo containing sand