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An exact solution for the nonlinear forced vibration of functionally graded nanobeams in thermal environment based on surface elasticity theory

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ABSTRACT

In the present investigation, an exact solution is proposed for the nonlinear forced vibration analysis of nanobeams made of functionally graded materials (FGMs) subjected to thermal environment including the effect of surface stress. The material properties of functionally graded (FG) nanobeams vary through the thickness direction on the basis of a simple power law. The geometrically nonlinear beam model, taking into account the surface stress effect, is developed by implementing the Gurtin–Murdoch elasticity theory together with the classical Euler–Bernoulli beam theory and using a variational approach. Hamilton's principle is utilized to obtain the nonlinear governing partial differential equation and corresponding boundary conditions. After that, the Galerkin technique is employed in order to convert the nonlinear partial differential equation into a set of nonlinear ordinary differential equations. This new set is then solved analytically based on the method of multiple scales which results in the frequency–response curves of FG nanobeams in the presence of surface stress effect. It is revealed that by increasing the beam thickness, the surface stress effect diminishes and the maximum amplitude of the stable response is shifted to the higher excitation frequencies.

1. Introduction

In recent years, the rapid technological developments in nanoscience and engineering and the achievements in fabrication and manufacturing bring us more and more nano- and micro-electro-mechanical systems. Among these micro/nano structures, nanobeams have been extensively used in many applications of nano-sized devices and systems [1–6]. Moreover, the design of nanobeam is dominated by various basic requirements. One of these basic requirements is to achieve nonlinear dynamics properties to match the desired functionality.

The free vibration analysis of structural elements is a common study as important as among all engineering problems and knowledge of the natural frequencies suggests the designer avoid the peak resonances which occur nearby the natural frequencies. Furthermore, time-dependent external forces lead to forced vibration in dynamic systems and analyzing the response of structure around resonance condition is necessary in this case.

To incorporate the quantum effects, that exist at nanoscale, into classical continuum theory, the classical continuum needs to be refined. Modified continuum models are one of the most applied theoretical approaches for the investigation of nanomechanics due to their computational efficiency and the capability to produce accurate results which are comparable to the atomistic models ones. As examples of using non-classical continuum mechanics in forced vibration analysis of beam at small scales, Uymaz [7] considered a forced vibration analysis of functionally graded nanobeam based on the size-dependent nonlocal continuum elasticity. Ghayesh et al. [8] investigated the nonlinear forced vibration of microbeams employing the size-dependent strain gradient continuum elasticity.

However, one of the most important size dependency of nanostructures is the effect of surface stress which can be easily observed at the atomic scale due to high surface to volume ratio, and this has been clearly indicated and explained [9,10]. The main reason of the phenomenon is related to the different environment conditions for the atoms which their positions are near free surface compared to ones at the bulk of the material. Therefore, in order to take surface stress effect into account, the classical continuum theory needs to be modified. For this purpose, Gurtin and Murdoch [11,12] developed a theoretical concept on the basis of the continuum mechanics including surface stress effects, in which a surface is regarded as a mathematical deformable layer of zero thickness with different material properties from the underlying bulk and completely adhered to the underlying bulk material. Later, Lu et al. [13] improved

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Gurtin–Murdoch model by assuming linear variation through the thickness of the normal stress inside and on the surface of bulk substrate to satisfy the constitutive relations. Afterward, the proposed improved model has been employed in different studies to indicate the effect of surface stress on mechanical characteristics of nanostructures.

For example, Wang and Feng [14] examined the effect of surface stress on the nanosized contact problems and they concluded that both of the indent depth and the maximum normal contact stress depend strongly on the surface stress of nanoindentation. The classical generalized shear deformable theory was adopted to model the film bulk. Ricci and Ricciardi [15] developed a new finite element approach to investigate the surface stress effect on mechanical behaviors of microstructures using the standard form of principle of virtual work. Ansari and Sahmani [16] presented non-classical beam model models based on Gurtin–Murdoch elasticity theory and different beam theories to analyze bending and buckling behaviors of nanobeams including surface stress effect. Ansari and Sahmani [17] studied the free vibration response of nanoplates including surface stress effects based on the continuum modeling approach. They implemented the Gurtin–Murdoch elasticity theory into the different types of plate theory. Fu and Zhang [18] predicted the pull-in voltages of electrically actuated nanobeams incorporating surface energies. Ansari et al. [19] investigated the surface stress effect on the free vibration response of circular nanoplates subjected to various edge supports based on the Gurtin–Murdoch elasticity theory and first-order shear deformation plate theory. Shaat et al. [20] investigated the size-dependent bending behavior of ultra-thin functionally graded (FG) Mindlin nanoplates by incorporating surface stress effect into the conventional linear Mindlin plate theory. Also, the effect of surface stresses on the bending of functionally graded nano-scale films using finite element method based on the Mindlin plate theory is studied by Shaat et al. [21]. The size-dependent the bending and resonance behavior of nanowires based on the based on Timoshenko beam theory considering high-order surface stress effects studied by Chiu and Chen [22]. Recently, Ansari et al. [23,24] applied the Gurtin–Murdoch elasticity theory to Euler–Bernoulli and Timoshenko beam theories, respectively, to predict postbuckling behavior of nanobeams in the present of surface stress effect. Also, Malekzadeh et al. [25] performed the nonlinear free flexural vibration of skew nanoplates by considering the influences of free surface energy and size effect.

In the present work, the main goal is to examine the effect of surface stress on the nonlinear forced vibration characteristics of nanobeams made of functionally graded materials (FGMs) subjected to thermal environment. To this end, the Gurtin–Murdoch continuum elasticity is used with the classical Euler–Bernoulli beam theory to develop a non-classical beam model taking into account the effect of surface stress. The method of multiple scales in conjunction with the Galerkin technique is utilized to present an exact solution for the nonlinear governing differential equation.

2. Theoretical formulations of a continuum beam model incorporating surface effects

As depicted in Fig. 1, an FG nanobeam of length L and thickness h that is made from a mixture of ceramics and metals is considered. It is assumed that the materials at bottom surface ($z = -h/2$) and top surface ($z = h/2$) of the microbeam are metals and ceramics, respectively. The effective material properties of the FG nanobeam such as Young's modulus (E), mass density (ρ), Poisson's ratio (ν), thermal expansion coefficient (α), thermal conductivity (K), surface Lame's constants (λ^s and μ^s), surface residual stress (τ^s), and surface mass density (ρ^s) can be determined as following

$$E(z)=(E_m-E_c)V_f(z)+E_c,\ \ \rho(z)=\left(\rho_m-\rho_c\right)V_f(z)+\rho_c,\nu(z)=(\nu_m-\nu_c)V_f(z)+\nu_c,$$
$$\alpha(z)=(\alpha_m-\alpha_c)V_f(z)+\alpha_c,\ \ K(z)=(K_m-K_c)V_f(z)+K_c \tag{1a}$$

$$\lambda^s(z)=\left(\lambda^s_m-\lambda^s_c\right)V_f(z)+\lambda^s_c,\ \ \mu^s(z)=\left(\mu^s_m-\mu^s_c\right)V_f(z)+\mu^s_c,$$
$$\tau^s(z)=\left(\tau^s_m-\tau^s_c\right)V_f(z)+\tau^s_c,\ \ \rho^s(z)=\left(\rho^s_m-\rho^s_c\right)V_f(z)+\rho^s_c \tag{1b}$$

The subscripts m and c denote metal and ceramic phases, respectively. Various types of functions can be used to describe the variation of the volume fraction of constituents. Here a simple power law function is considered as following to describe the volume fraction as below

$$V_f(z)=\left(\frac{1}{2}+\frac{z}{h}\right)^k \tag{2}$$

where k is the volume fraction exponent.

Also, it is assumed that the ceramic-rich and metal-rich surfaces have the temperature values of T_c and T_m, respectively. The temperature distribution can be obtained by solving the following heat conduction equation for the given boundary conditions

$$K\frac{d^2T}{dz^2}=0\,,\quad T\left(\frac{h}{2}\right)=T_m,\quad T\left(-\frac{h}{2}\right)=T_c \tag{3}$$

Applying Eq. (3) along the beam thickness results in a linear temperature distribution as follows

$$T=\left(\frac{T_m+T_c}{2}\right)+(T_m-T_c)\frac{z}{h} \tag{4}$$

2.1. Kinematics and constitutive relations

On the basis of the Euler–Bernoulli beam theory, the displacement field at any point (x,y,z) and at any time $t$can be introduced as

$$u_x=U_0(x,t)-\bar{z}\frac{\partial W(x,t)}{\partial x},\ u_y=0,\ u_z=W(x,t) \tag{5}$$

in which U_0 and Wstand for the displacement of neutral axis in x direction and the lateral deflection, respectively.

By assuming small slopes in the beam after deformation, the axial strain can be approximately given by the von-Karman strain as

$$\varepsilon_{xx}=\frac{\partial U_0}{\partial x}-\bar{z}\frac{\partial^2 W}{\partial x^2}+\frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2=U_0'-\bar{z}W''+\frac{1}{2}(W')^2 \tag{6}$$

where the prime symbol refers to the derivative with respect to x.

The non-zero component of the Cauchy stress tensor can be obtained as

$$\sigma_{xx}=(\lambda+2\mu)\left(U_0'-\bar{z}W''+\frac{1}{2}(W')^2\right)-\beta\Delta T \tag{7}$$

where $\lambda=E\nu/(1-\nu^2)$ and $\mu=E/(2(1+\nu))$ are Lame constants, the parameter $\beta=\alpha E/(1-\nu)$ is the stress–temperature modulus and α is thermal expansion coefficient and $\Delta T = T-T_0$, where $T$is the temperature distribution through the FG beam and T_0 is reference temperature and the position of neutral line z_0 can be obtained by the following equation

$$z_0=\frac{\int_A z(\lambda(z)+2\mu(z))dA}{\int_A(\lambda(z)+2\mu(z))dA} \tag{8}$$