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Wavelet-based finite element method for multilevel local plate analysis

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ABSTRACT

In this paper, an efficient multilevel method is presented for local static analysis of plates based on the coupling of finite element method and discrete wavelet transform (FEM–DWT). The problem is discretized using finite element method and the corresponding governing equation is transformed into a localized one by applying the discrete Haar wavelet. Then the obtained governing equation is reduced using special averaging and reduction algorithms. Two types of the localization approach are used, the first is localization with respect to nodes and it is rather efficient for plates with localized parameters (such as concentrated loads or stress concentration). Another approach is the localization with respect to the degrees of freedom of each node and it is rather efficient for the evaluation of the effect of the degrees of freedom on the plate. The numerical results indicate that the proposed method provides accurate results for the selected regions, with respect to corresponding FEM solution, with a considerable reduction in the size of the problem. Moreover, in problems which itself have a localized properties, the efficiency of the method is considerably increased.

1. Introduction

Plates are initially flat structural members bounded by an edge or boundary, which their thickness is small compared with other characteristic dimensions of the faces (length, width, diameter, etc.) [1]. The static or dynamic loads carried by plates are predominantly perpendicular to the plate faces. The load-carrying action of a plate is similar, to a certain extent, to that of beams or cables; thus, plates can be approximated by a grid work of an infinite number of beams or by a network of an infinite number of cables, depending on the flexural rigidity of the structures. This two-dimensional structural action of plates results in lighter structures, and therefore offers numerous economic advantages. The flat plate develops shear forces, bending and twisting moments to resist transverse loads. The loads are generally carried in both directions and the twisting rigidity in isotropic plates is quite significant, therefore, a plate is considerably stiffer than a beam of comparable span and thickness. Consequently, thin plates combine lightweight and a form efficiency with high load-carrying capacity, economy, and technological effectiveness. Because of the distinct advantages discussed above, thin plates are extensively used in all fields of engineering. Plates are used in architectural structures, bridges, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments, machine parts, and other structural components [2–4]. One of the more efficient method for the solution of the plate problems is the finite element method (FEM) [5–12]. Finite element method has emerged as a very efficient mathematical tool in engineering applications. The static analysis of the problems by the FEM led to the formation of resolving a system of linear algebraic equations (SLAE) with an immense number of unknowns [13,14]. Generally, this is the most time-consuming stage of the computing [15,16], especially if we take into account the limitation in the power of the contemporary software and in the performance of personal computers or even advanced supercomputers and the necessity to obtain correct and accurate solution in a reasonable time. However, practically in many cases it is unreasonable or impossible to obtain such solutions for the entire structure and due to structural or loading conditions, the location and approximate dimensions of critical and most vital for designers regions of the structure can be determined. The stress–strain state in these regions is of paramount importance from the standpoint of analysis and design, and may lead to structural failure or cause impairment in structural performance [17]. On the other hand, many problems itself have localized properties or there is a need for the obtaining the solution of especial zones of the plate and the specified localization of the problem can be more efficient (by considering of the computational efforts).

Local mesh refinement in the finite element method is one of the approaches to obtaining accurate results in these critical zones. In other words, find a mesh, which led to few degrees of freedom

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as possible with the controlled error in the corresponding finite element solution. This method called the adaptive finite element method [18,19], and aims at economical computation of arbitrary quantities of physical interest by properly adapting the computational mesh [20]. Another method, which was proposed by Fish et al. [21] is a multilevel finite element approach with a superposition technique for improving the quality of numerical solutions and mathematical models of a certain class of problems. This method was presented as an attempt to construct a nearly optimal discretization scheme and improve the quality of the solution without changing the mesh size by superimposing a sequence of overlapping finite element meshes on the portion(s) of the initial finite element mesh. All of the above-mentioned methods, attempted to reduce the computational works by obtaining an optimal mesh. Another multilevel approach for structural analysis, which is applied after FEM discretization, is a coupling of solution method with multilevel mathematical tools. Wavelet analysis is a powerful computational-analytical tool for the decomposition and multilevel mathematical modeling of functions and solution of boundary value problems [22–26]. Wavelet has extraordinary characteristics and combines the advantages of functional analysis [27], Fourier transform [28], spline analysis [29], harmonic analysis [30] and numerical analysis [31] as well, and can be successfully employed for the considering goal. In this approach, after discretization and obtaining of governing equations, the considering problem is transformed into a multilevel space by multilevel wavelet transform. In recent years, Akimov et al. [32–37] have developed this multilevel analysis approach by combining so-called discrete-continual finite element method (DCFEM) [38] and discrete wavelet transform (DWT) [39]. This method is applicable for structures, which have constant, piecewise constant or in general regular physical and geometrical parameters along one of the coordinate's directions (so-called “basic” direction). In addition, for the solution of the simplest problem of local static analysis of Bernoulli Beam on elastic foundations [40,41] authors used finite difference method [42] and discrete wavelet transform and obtained local accurate results.

This paper is devoted to local analysis of plates, which is provided by coupling of FEM and DWT. Corresponding so-called FEM–DWT method reduces the size of the problem also provide the accurate results in selected regions simultaneously. This is a rather efficient approach for evaluation of local phenomenon such as stress concentration or concentrated force. FEM–DWT solution provides the qualitative and quantitative assessments of the degree of localization of various kinds of design factors and the evaluation of the effect of each degree of freedom on the behavior of the plate. The efficiency of the computational complexity (number of operations) of the proposed method can be evaluated by the comparison of unreduced (n) and reduced (n_r) total number of degrees of freedom of the structure. Let, $N_{comp} = O(n^3)$ and $N^r_{comp} = O(n_r^3)$ are the approximate computational complexity of the FEM and FEM–DWT (without the considering of bandwidth of the stiffness matrix), respectively. Then, the comparative reduction in the number of the iteration can be approximated by n^3/n_r^3. The paper is organized as follows. In the next section, the general finite element formulation of the problem and theoretical basis of the transition to wavelet basis is presented. The transition of the problem into Haar wavelet basis is described, in details, in Section 3. Theoretical basis of reduction in size of wavelet representation is presented in Section 4. Various types of localization process are described in Section 5. Subsequently, the efficiency, accuracy and validity of the proposed method are demonstrated by several numerical examples in Section 6. Finally, some concluding remarks are presented.

2. Formulation of the problem and theoretical basis of the transition to wavelet basis

Consider the boundary value problem described by the following equation

$$L\bar{u} = \bar{F} \tag{1}$$

where L is the operator of the boundary value problem formulated by taking into account the boundary conditions in the framework of the standard (extended) area proposed by Zolotov [43]; $\bar{u}$ is the unknowns vector and $\bar{F}$ is the given vector of right hand side of the problem. Then the Formulation (1) corresponds to the below energy functional:

$$\Phi(\bar{u}) = 0.5\cdot(L\bar{u},\, \bar{u}) - (\bar{F},\, \bar{u}) \tag{2}$$

which is the stationary point of those, is the solution of (1). In the Eq. (2) the $(\bar{f}\,,\, \bar{g})$ is the scalar product of the functions $\bar{f}$ and $\bar{g}$ [44].

Let the following form as a finite element method approximation of the problem:

$$K\bar{u}_n = \bar{f}_n \tag{3}$$

where K is the finite element analog of the original operator of the continuum formulation (1) or the stiffness matrix of the problem, $\bar{u}_n = [u_1 u_2 ... \, u_n\,]^T$ is the discrete (approximated) unknowns vector, $\bar{f}_n = [f_1 f_2 ... \, f_n\,]^T$ is the discrete (approximated) given vector of right hand side of the problem and n is the dimension of the discrete problem (number of the degrees of freedom of the problem). Using the functional (2) and the finite element approximation (3), the transition from the original unit basis to the Haar basis is as following:

$$\begin{aligned} \Phi(\bar{u}_n) &= 0.5\cdot(K\bar{u}_n,\, \bar{u}_n) - (\bar{f}_n,\, \bar{u}_n) \\ &= 0.5\cdot(KQ\bar{v}_n,\, Q\bar{v}_n) - (\bar{f}_n,\, Qv_n) = 0.5\cdot(Q^*KQv_n,\, v_n) - (Q^*\bar{f}_n,\, \bar{v}_n), \end{aligned} \tag{4}$$

$$\tilde{\Phi}(\bar{v}_n) = 0.5\cdot\left(Q^*KQ\bar{v}_n,\, \bar{v}_n\right) - \left(Q^*\bar{f}_n,\, \bar{v}_n\right) \tag{5}$$

where Q is a unnormalized transition matrix comprising Haar basis vectors [45], $\bar{v}_n$ is the vector of discrete Haar basis coefficients as follows:

$$\bar{u}_n = Q\bar{v}_n \tag{6}$$

The problem can be rewritten with respect to a new unknown $\bar{v}_n$ in the following form:

$$\tilde{K}\bar{v}_n = \bar{F}_n; \quad \tilde{K} = Q^*KQ; \quad \bar{F}_n = Q^*\bar{f}_n \tag{7}$$

3. Discrete Haar wavelet

Due to the high efficiency of the localization process, the simplicity of the computational algorithm and its computer realization, the discrete Haar wavelet basis is used and corresponding direct and inverse algorithms of transformations are performed [39,46]. In the Haar wavelet, the basis functions are constructed by simple step functions and therefore have more computationl efficiency in the impelementation process.

3.1. Two-dimensional discrete Haar wavelet

Let $f(x_1, x_2)$ is an arbitrary two-dimensional function in region $\omega = \{(x_1, x_2)\colon 0 \le x_1 \le l_1,\, 0 \le x_2 \le l_2\}$ (Fig. 1). Where x_1, x_2 are coordinates; l_1, l_2 are dimensions along x_1, x_2. Then by dividing ω to $(n-1)$ part along x_1 and x_2 (where $n = 2^M$ is a number of discrete