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Shear effect on buckling of cellular columns subjected to axially compressed load

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ABSTRACT

This paper presents an analytical solution for calculating the critical buckling load of simply supported cellular columns when they buckle about the major axis. The solution takes into account the influence of web shear deformation on the buckling of cellular columns and is derived using the stationary principle of potential energy. The formula derived for calculating the critical buckling load is validated using finite element analysis results. It is shown from the present analytical solution that the web shear deformation can significantly reduce the buckling resistance of cellular columns. The influence of the shear deformation on the critical buckling load increases with the cross-section area of the tee section and the radius of circular holes but decreases with the length and the web thickness of the cellular column.

1. Introduction

Cellular and castellated columns and beams are widely used as structural members in buildings due to their large depth and light weight. A cellular/castellated column/beam is usually fabricated from a standard universal I-beam by using the process of a profiled flame cut along its length, then shafting into the profile, followed by the welding of the two halves together. This manufacturing process increases the beam's depth and thus the strength and stiffness associated with the bending about its major axis without adding additional material. Therefore, cellular and castellated beams are most often used in long span applications with light or moderate loadings such as primary beams in floors and roofs. A further advantage of the beams is the holes in their web which provide a route for services.

It is well known that the axially compressed buckling of columns can be evaluated using the classical Euler formula. However, the derivation of Euler formula considered only the flexural stiffness and deformations of the column but ignored the effect of shear deformations on the buckling load. Engesser [1,2] modified the Euler formula by considering the shear deformations in the plain-webbed column. The accuracy of Engesser's formula in considering the shear effect on the elastic stability of plain-webbed columns was verified by Nanni [3] through the application of the three-dimensional theory of elasticity. Later, Ziegler [4] provided more validation to Engesser's approach where he indicated the importance of considering shortening of the column on magnifying the shear effect on the buckling capacity. Using Engesser's formula, Gjelsvik [5] investigated the stability of columns with finite shear stiffness. He concluded that Engesser's method is valid for use with columns modeled as continuous Timoshenko shear beams in which plane sections are assumed to remain plane after deformation, but do not remain normal to the displaced axis of the column.

Unlike plain-webbed columns, shear deformations in built-up columns are more pronounced, which can significantly reduce the buckling capacity of the members [6]. In literature the effect of shear on the buckling capacity of built-up columns has been reported [7], which revealed that built-up columns exhibit reduced shear stiffness resulting in an increase in lateral deflection and consequently a reduction in the compressive load carrying capacity. This indicates that the buckling theory with taking into account shear deformations developed by Timoshenko and Gere [6] for plain-webbed columns may not be suitable for built-up columns. The axially compressed buckling problems of cellular and castellated columns were studied by Sweedan et al. [8] and El-Sawy et al. [9], respectively, in which the columns with various different boundary conditions due to shear and flexural deformations were investigated using finite element numerical analysis methods. The numerical results demonstrated the effect of web

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openings on the critical buckling loads of the columns when they buckle about their major axis. Recently, analytical solutions were developed by Chen and Li [10] for predicting the critical buckling load of battened columns and by Yuan et al. [11] for predicting the critical buckling load of castellated columns. Apart from the above-mentioned work, which is directly related to the axially compressed stability of columns with shear effect, considerable amount of work also exists on web post buckling [12], lateral torsional buckling [13], distortional buckling [14], interaction of different buckling modes [15], and nonlinear ultimate analysis [16,17] of castellated and cellular beams. The vibration problems of castellated beams with considering web shear effect were also discussed recently by Chen et al. [18] and Gu [19]. However, no analytical solution is available for the axially compressed buckling of cellular columns. Although the cellular column is similar to the castellated column in terms of web openings, they do have some difference. In the former the web openings can be adjustable, while in the latter, the web openings are not adjustable. In other words, the cellular column has more geometrical parameters than the castellated column does in the design. In this paper an analytical method is presented on the analysis of buckling of cellular columns. The analysis is accomplished using energy methods. A simple close-form solution for determining the critical buckling load of simply supported cellular columns subjected to an axial compression load is developed. The critical buckling load derived includes the influence of shear weakness due to web openings. The critical buckling load derived is validated using finite element analysis results.

2. Theoretical formulae

Consider a cellular column shown in Fig. 1a, in which the flange width and thickness are b_f and t_f, the web depth and thickness are h_w and t_w, the radius of circular holes is a, the distance between the centroids of the upper and bottom tee sections is $2e$, and the distance between the centers of two neighboring holes is b (see Fig. 1b). Let $u_1(x)$ and $u_2(x)$ be the axial displacements of the centroids of the upper and bottom tee sections, and $w(x)$ be the transverse displacement of the section (i.e. all points on a section have the same transverse displacement). Assume that, under the action of transverse loading, the two tee section beams behave as Bernoulli beam. According to the displacement assumptions used

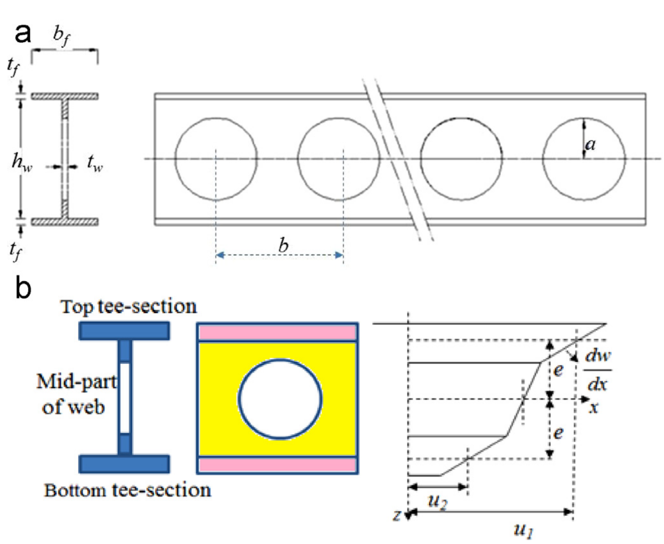

Fig. 1. (a) Definition of notations used in a cellular column. (b) Normal displacement distribution in the cross-section.

in Bernoulli beam, the axial displacement at any point in the tee sections is shown in Fig. 1b and can be expressed as follows:

For upper tee section:

$$u(x, z_1) = u_1(x) - z_1\frac{dw}{dx} \tag{1}$$

For bottom tee section:

$$u(x, z_2) = u_2(x) - z_2\frac{dw}{dx} \tag{2}$$

where z_1 and z_2 are the vertical axis in the local coordinates of the tee sections. The normal strains in the tee sections can be expressed as follows:

For upper tee section:

$$\varepsilon_x(x, z_1) = \frac{du_1}{dx} - z_1\frac{d^2w}{dx^2} \tag{3}$$

For bottom tee section:

$$\varepsilon_x(x, z_2) = \frac{du_2}{dx} - z_2\frac{d^2w}{dx^2} \tag{4}$$

The strain energy of the tee sections due to the axial and transverse displacements can be expressed as follows,

$$U_1 = \frac{EA_{tee}}{2}\int_o^l\left[\left(\frac{du_1}{dx}\right)^2 + \left(\frac{du_2}{dx}\right)^2\right]dx + EI_{tee}\int_o^l\left(\frac{d^2w}{dx^2}\right)^2 dx \tag{5}$$

where E is the Young's modulus, A_{tee} is the area of the tee-section, I_{tee} is the second moment of area of the tee-section about its own centroid axis, and l is the length of the column. Note that here the two tee sections are assumed to be identical. A_{tee} and I_{tee} are calculated as follows,

$$A_{tee} = b_f t_f + t_w\left(\frac{h_w}{2} - a\right) \tag{6}$$

$$I_{tee} = \frac{b_f t_f^3}{12} + b_f t_f\left(\frac{h_w + t_f}{2} - e\right)^2 + \frac{t_w}{12}\left(\frac{h_w}{2} - a\right)^3 + t_w\left(\frac{h_w}{2} - a\right)\left(\frac{h_w + 2a}{4} - e\right)^2 \tag{7}$$

Assume that the bending moment is taken only by the two tee sections and the shear force is taken only by the web posts. The shear displacement of the web posts can be expressed as follows,

$$\Delta = u_1(x) - u_2(x) - 2e\frac{dw}{dx} \tag{8}$$

The shear strain energy of the web posts can be expressed as follows,

$$U_2 = \sum\frac{1}{2}k_{sh}\Delta^2 \approx \frac{1}{2b}\int_o^l k_{sh}\Delta^2 dx \tag{9}$$

where k_{sh} is the shear stiffness of the web post. The summation in Eq. (9) applies to all web posts. The conversion of summation to integration made in Eq. (9) is due to the use of smear model.

In order to determine the shear stiffness, a unit shear force is applied on the top of the web post (see Fig. 2). The shear stress, τ, and shear strain, γ, in the web post thus can be expressed as follows,

$$\tau(z_3) = \frac{1}{t_w\left(b - 2\sqrt{a^2 - z_3^2}\right)} \tag{10}$$