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Free vibration analysis of simply supported beams with solid and thin-walled cross-sections using higher-order theories based on displacement variables

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ABSTRACT

Solutions for undamped free vibration of beams with solid and thin-walled cross-sections are provided by using refined theories based on displacement variables. In essence, higher-order displacement fields are developed by using the Carrera unified formulation (CUF), and by discretizing the cross-section kinematics with bilinear, cubic and fourth-order Lagrange polynomials. Subsequently, the differential equations of motion and the natural boundary conditions are formulated in terms of fundamental nuclei by using CUF and the strong form of the principle of virtual displacements. The second-order system of ordinary differential equations is then reduced into a classical eigenvalue problem by assuming simply supported boundary conditions. The proposed methodology is extensively assessed for different solid and thin-walled metallic beam structures and the results are compared with those appeared in published literature and also checked by finite element solutions. The research demonstrates that: (i) the innovative 1D closed form CUF represents a reliable and compact method to develop refined beam models with solely displacement variables; (ii) 3D-like numerically exact solutions of complex structures can be obtained with ease; and (iii) the numerical efficiency of the present method is uniquely robust when compared to other methods that provide similar accuracies.

1. Introduction

The undamped free vibration analysis of structures has always been a major area of activity in structural design. The results of modal analyses are, in fact, of great interest in dynamic response analyses, acoustics, aeroelasticity and also to avoid resonance. Even today, for a certain class of structures, the most convenient way of conducting modal analyses is by means of idealizing the structure by simplified beam models. Beam models are easy to use and important tools for structural analysts. In aircraft structural design, for example, such models are still widely used in the modelling of helicopter rotor blades, aircraft wings, and propeller blades, amongst others.

The classical and oldest one-dimensional (1D) beam theory is that of Euler [1] and Bernoulli [2], hereinafter referred to as EBBM (Euler–Bernoulli beam model), which underwent further developments by Saint-Venant [3,4] and Timoshenko [5,6], hereinafter referred to as TBM (Timoshenko beam model). As it is well known, the EBBM does not account for transverse shear deformations, while the TBM incorporates a uniform shear distribution along the cross-section of the beam (see more details in [7]). However, these classical beam models have severe limitations (e.g., the impossibility of dealing with constrained warping and shear-bending couplings). Thus, there are several problems in the engineering practice that cannot be solved with these traditional tools. Deep and thin-walled beams are some examples for which advanced treatment might be necessary.

Many refined beam models can be found in the literature which overcome the shortcomings of classical models. A comprehensive review about existing beam and plate theories was published by Kapania and Raciti [8,9], who investigated the vibrations, wave propagation, buckling and post-buckling behaviors. Another review about modern theories for beam structures was recently published by Carrera et al. [10]. However, a brief overview about refined 1D models is given here for the sake of completeness. Particular attention should be paid to the pioneering works by Sokolnikoff [11] and Timoshenko and Goodier [12]. Gruttmann et al. [13–15] computed shear correction factors for torsional and

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flexural shearing stresses in prismatic beams, arbitrary shaped cross-sections as well as wide- and thin-walled structures. The 3D elasticity equations based on Saint-Venant solution were reduced to beam-like structured by Ladevéze et al. [16–18] for high aspect ratio beams with thin-walled sections. Yu et al. [19–21] used the variational asymptotic solution of beams to build an asymptotic series. To enhance the description of the normal and shear stress of the beam, El Fatmi [22,23] introduced improvements of the displacement models over the beam section by introducing a warping function. With the advent of the finite element method (FEM), various beam models were developed for validation purposes and a considerable overview was provided by Reddy [24,25], whose works discussed both classical and higher-order 1D elements, together with the problem of shear-locking.

As far as the free vibration analysis is concerned, Eisenberger et al. [26] presented a method to compute the exact vibration frequencies of asymmetrical laminated beams. Three higher-order models to analyze the free vibrations of deep fiber reinforced composite beams were addressed by Marur and Kant [27], and the same authors extended this theory to study vibrations of angle-ply laminated beams by accounting for transverse shear and normal strain effects [28]. A higher order finite element model based on the classical lamination theory was developed by Ganesan and Zabihollah (see [29,30]), and vibration response from laminated tapered composite beams was subsequently investigated. Kameswara et al. [31] studied a closed form solution with high-order mixed theory for free vibration analysis of composite beams. Numerical examples were computed for beams of various span to height ratios, and the results showed that their theories provide lower natural frequencies than those computed through Timoshenko model in case of thick sandwich beams.

All the publications mentioned above show that refined beam theories and the vibration analysis of slender structures still attract considerable attention of researchers and engineers. The current work presents a new method to deal with the free vibration behavior of beam structures. This method is based on the well-known Carrera unified formulation (CUF), which was introduced by Carrera et al. [32–34] for plates and shells. CUF was extended to beam structures by Carrera and Giunta [35] in 2010. Since then, various improvements of CUF have taken place and a brief overview is given below. The strength of CUF is that it allows the automatic development and compact formulation of any theory of structures by expressing the 3D displacement field as an expansion series of the generalized unknowns, which lie on the beam axis in the case of 1D models, through certain cross-sectional functions (see [7] for a comprehensive discussion about CUF). Several papers in the literature made use of Taylor series polynomials as cross-sectional functions, and the corresponding models were referred to as TE (Taylor expansion). TE models have demonstrated higher-order capabilities in dealing with various beam problems both in conjunction with FEM methods [36–38] and exact solutions [39–41]. More recently, Carrera and Petrolo [42,43] adopted the Lagrange polynomials to discretize the cross-sectional kinematics and the resulting LE (Lagrange expansion) CUF models have been successfully used for the analysis of both metallic and laminated composite structures. Some of the main advantages of the LE models are that they only involve pure displacement unknowns and allow the component-wise analysis of complex structures [44].

In the previous literature about LE models, FEM was applied to solve the weak form governing equations. In the present work, for the first time, numerically exact solutions of the strong-form equations of motion of LE models for the free vibration analyses of solid and thin-walled structures are presented by assuming simply supported boundary conditions. The present methodology is said to be exact in the sense that it provides exact solution of the equations of motion of a structure once the initial assumptions on the displacement field have been made.

The paper is organized as follows: (i) First, the adopted notation and some preliminary relations are introduced in Section 2. (ii) CUF is then presented in Section 3, along with LE models for beams. (iii) Next, the governing differential equations and natural boundary conditions are derived in Sections 4 and 5. Here, by adopting simply supported boundary conditions, the differential problem is reduced into a linear eigenvalue problem in terms of CUF fundamental nuclei. (iv) Subsequently, a number of significant problems are treated in Section 6. (v) Finally, the main conclusions are outlined.

2. Preliminaries

The coordinate frame of the generic beam model is shown in Fig. 1. The beam has cross-section Ω and length L. The dimensions along y are $0 \le y \le L$. The displacement vector is

$$\mathbf{u}(x, y, z; t) = \{ u_x \; u_y \; u_z \}^{\mathrm{T}} \tag{1}$$

in which u_x, u_y and u_z are the displacement components along x-, y- and z-axes, respectively. The superscript "T" represents a transpose. The stress, $\boldsymbol{\sigma}$, and the strain, $\boldsymbol{\epsilon}$, components are grouped as follows:

$$\boldsymbol{\sigma} = \{ \sigma_{yy} \; \sigma_{xx} \; \sigma_{zz} \; \sigma_{xz} \; \sigma_{yz} \; \sigma_{xy} \}^{\mathrm{T}}, \quad \boldsymbol{\epsilon} = \{ \epsilon_{yy} \; \epsilon_{xx} \; \epsilon_{zz} \; \epsilon_{xz} \; \epsilon_{yz} \; \epsilon_{xy} \}^{\mathrm{T}} \tag{2}$$

In the case of small displacements with respect to a characteristic dimension in the plane of Ω, the strain–displacement relations are

$$\boldsymbol{\sigma} = \mathbf{D}\mathbf{u} \tag{3}$$

where $\mathbf{D}$ is the following linear differential operator matrix:

$$\mathbf{D} = \begin{bmatrix} 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \tag{4}$$

According to Hooke's law, the relationship between stress and strain is

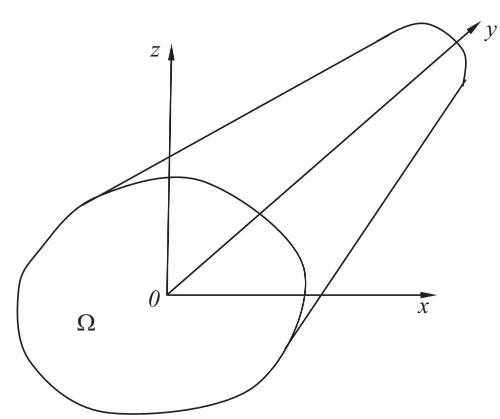

Fig. 1. Beam model and related Cartesian frame.