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Short communication

Free vibration analysis of a nonlinear panel coupled with extended cavity using the multi-level residue harmonic balance method

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ABSTRACT

This article addresses the free vibration analysis of a nonlinear panel coupled with extended cavity. In practice, the cavity length of a panel-cavity system is sometimes longer than the panel length. Therefore, this study examines the effect of cavity length on the natural frequency of a nonlinear panel coupled with extended cavity. The multi-level residue harmonic balance method, which was recently developed by the author and his research partners, is used to solve this nonlinear problem. The present harmonic balance solution agrees reasonably well with the results obtained from a previous classical solution and shows that the cavity length is a very important factor that significantly affects the panel vibration and should not be ignored in the modelling process. The natural frequency of a panel-cavity system is very sensitive to the cavity length and decreases significantly when the cavity is longer, due to its larger volume or weaker stiffness. Moreover, when the cavity is very long and its resonant frequencies are close to that of the panel, multiple frequency solutions are obtained.

1. Introduction

Over the past few decades, numerous studies on panel-cavity coupling have been published (e.g., Sadri and Younesian [1] and Hui et al. [2]). To the best of the author's knowledge, almost all of these adopted a model in which the cavity length is equal to the panel length, although in practice, it is sometimes longer. Fig. 1 shows two examples – (1) the clamping fixture occupies some spaces and (2) the nine individual panels are mounted on the clamping grids and share one common cavity – that motivated this study on the effect of cavity length. Moreover, studies of this nonlinear structural-acoustic problem are still limited, although many nonlinear panel or linear structural-acoustic problems have been solved (e.g., Younesian et al. [3,4] and Shi et al. [5]). In addition, the author and his research partners [6] recently developed the multi-level residue harmonic balance method to solve nonlinear beam/plate problems. The main advantage of this method is that only one set of nonlinear algebraic equations is generated in the zero-level solution procedure, while higher-level solutions to any desired degree of accuracy can be obtained by solving a set of linear algebraic equations. Table 1 compares the performance of the classical harmonic method and the multi-level residue harmonic balance method in solving cubic nonlinear beam problems. The number of nonlinear algebraic equations generated in the solution processes in the proposed method is clearly much smaller than that in the classical harmonic method, showing that it is time saving and less complicated.

2. Structural-acoustic formulation

Fig. 1 shows a nonlinear panel coupled with extended cavity. The cavity length is longer than the panel length. The acoustic pressure within the cavity induced by the panel in Example I, or in the centre panel in Example II is given by the following homogeneous wave equation (Lee and Ng [7]).

$$\nabla^2 P^h - \frac{1}{C_a^2}\frac{\partial^2 P^h}{\partial t^2} = 0, \tag{1}$$

where P_h is the acoustic pressure induced by the h-th harmonic component of the nonlinear panel vibration and C_a is the speed of sound. It is assumed, in Example II of Fig. 1, that the vibrations of the individual panels are not coupled with each other or that the vibration of one panel does not affect the others. The boundary conditions of the cavity are at $x=0$ and a

$$\frac{\partial P^h}{\partial x} = 0 \tag{2a}$$

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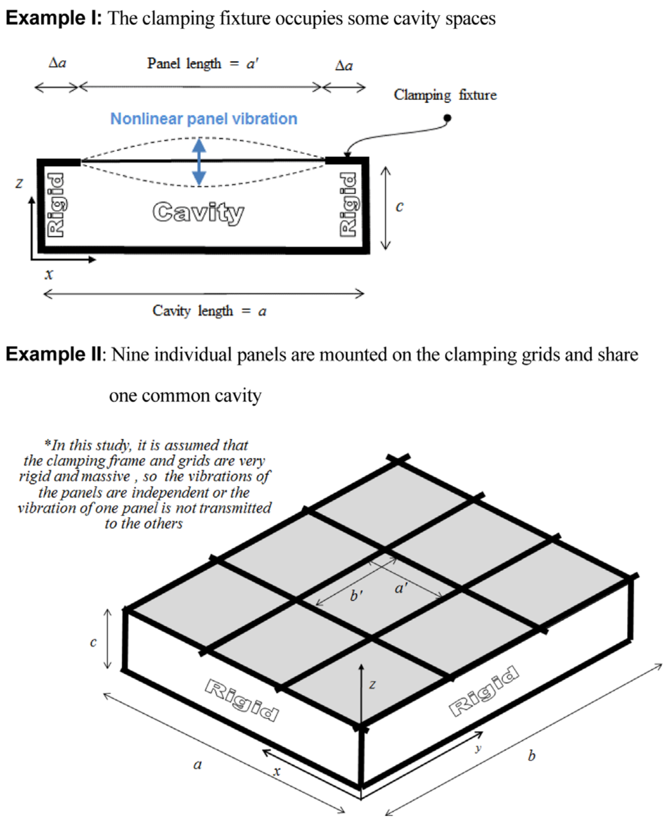

Fig. 1. Examples of a panel coupled with extended cavity.

at $y=0$ and b

$$\frac{\partial P^h}{\partial y}=0 \tag{2b}$$

at $z=0$

$$\frac{\partial P^h}{\partial z}=0 \tag{2c}$$

at $z=c$, for $\Delta a + a' > x > \Delta a$, and $\Delta b + b' > y > \Delta b$, $\frac{\partial P^h}{\partial z} = -\rho_a \frac{\partial^2 w^h}{\partial t^2}$

otherwise, $\frac{\partial P^h}{\partial z}=0$ (2d)

where a" and b'=panel width and length; a, b, and c=cavity length, width, and depth; and ρ_a=air density. w^h is the hth harmonic components of the nonlinear panel displacement at $z=c$, which is given in the following form

$$w^h = A^h(t)\phi(x, y) \tag{3}$$

where $A^h(t)$ is the modal amplitude while $\phi(x,y)$ is the mode shape (for a simply supported panel, $\phi(x, y) = \sin(\frac{\pi(x-\Delta a)}{a'})\sin(\frac{\pi(y-\Delta b)}{b'})$).

The total panel vibration is the summation of all harmonic components and is given by

$$w^h = \sum_{h=1,3,5\ldots}^{H} w^h \tag{4}$$

According to a previous study (Lee and Ng, 1998) [7] the general multi-acoustic mode solution of Eq. (1) is

$$P^h(x, y, z, t) = \sum_u^U \sum_v^V \left(L_{uv}^h \sinh\left(\mu_{uv}^h z\right) + N_{uv}^h \cosh\left(\mu_{uv}^h z\right)\right)\varphi_{uv}(x, y)T(t) \tag{5}$$

where $\mu_{uv}^h = \sqrt{(\frac{u\pi}{a})^2 + (\frac{v\pi}{b})^2 - (\frac{h\omega}{C_a})^2}$; $\phi_{uv}(x, y) = \cos(\frac{u\pi}{a}x)\cos(\frac{v\pi}{b}y)$ is the acoustic mode; u and v are the acoustic mode numbers; and ω is the excitation frequency. L_{uv}^h and N_{uv}^h are coefficients that depend on the boundary conditions at $z=0$ and $z=c$; U and V are the numbers of acoustic mode numbers used; and $T(t)$ is the time function.

By applying the boundary conditions in Eqs. (2c–d) to Eq. (5), the unknown modal coefficients, L_{uv}^h and N_{uv}^h can be expressed in terms of $A^h(t)$, and thus the hth harmonic component of the modal acoustic pressure force acting on the panel is given by

$$P_c^h(t) = \frac{\int_0^b \int_0^a P^h(x, y, z, t)\phi(x, y)dxdy}{\int_0^b \int_0^a \phi(x, y)^2 dxdy}$$

$$\Rightarrow$$

$$P_c^h(t) = K^h A^h(t) \tag{6}$$

where $K^h = -\sum_u^U \sum_v^V \frac{\rho_a (h\omega)^2}{\mu_{uv}^h} \frac{(\alpha_{uv}^{\phi})^2}{\alpha_{uv}^{uv}\alpha} \cot h(\mu_{uv}^h c)$; $\alpha_{uv}^{uv} = \int_0^b \int_0^a \varphi_{uv}\varphi_{uv} dxdy$; $\alpha_{uv}^{uv} = \int_0^b \int_0^a \phi^2 dxdy$; $\alpha_{uv}^{\phi} = \int_0^b \int_0^a \varphi_{uv}\phi dxdy$;
Then, the total modal acoustic pressure force at $z=c$ is given by

$$P_c = \sum_{h=1,3,5\ldots}^{H} P_c^h \tag{7}$$

In this study, the governing equation of nonlinear panel adopted by Hui et al. [2] is used here and is incorporated with the acoustic force term in equation (7).

$$\rho\frac{d^2A}{dt^2} + \rho\omega_o^2 A + \beta A^3 + P_c = 0$$

$$\Rightarrow$$

$$\rho\frac{d^2A}{dt^2} + \rho\omega_o^2 A + \beta A^3 + \sum_{h=1,3,5\ldots}^{H} K^h A^h = 0 \tag{8}$$

where $\omega_o = \sqrt{\frac{E\tau^2}{12\rho(1-\nu^2)}}((\frac{\pi}{a'})^2 + (\frac{\pi}{b'})^2)$=the fundamental linear natural frequency of the panel; $\beta = \frac{E\tau}{12(1-\nu^2)}\frac{\gamma}{(a')^4}$=nonlinear stiffness coefficient, $\gamma = 3\pi^4[(\frac{3}{4} - \frac{\nu^2}{4})(1 + r^4) + \nu r^2]$, $r = a'/b'$=aspect ratio; E=Young's modulus; ν=Poisson's ratio; ρ=density per unit

Table 1
Comparison between the numbers of algebraic equations generated in the multi-level residue harmonic balance method and classical harmonic balance method for cubic nonlinear beam problem.

	Zero level (one harmonic term)		1st level (two harmonic terms)		2nd level (three harmonic terms)	
	Nos. of nonlinear algebraic equations	Nos. of linear algebraic equations	Nos. of nonlinear algebraic equations	Nos. of linear algebraic equations	Nos. of nonlinear algebraic equations	Nos. of linear algebraic equations
Multi-level residue harmonic balance method	1	0	1	2	1	3
Classical harmonic balance method	1	0	2	0	3	0