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Shear buckling of rotationally-restrained composite laminated plates

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ABSTRACT

Based on the Galerkin method, a semi-analytical solution for the shear buckling of composite laminated plates with all four edges elastically-restrained against rotation is presented. The considered laminated plates are loaded in pure shear or combined shear and compression. The deformation shape function is constructed through a unique weighting combination of vibration eigenfunctions of simply-supported and clamped conditions, and it is an effective method for solving the considered eigenvalue problem. A parametric study is conducted to evaluate the effect of the rotational restraint stiffness, plate aspect ratio, material orthotropy and anisotropy on the buckling behavior of the rotationally-restrained laminated plates under pure shear or combined shear and compression action. The semi-analytical solution presented can be used to analyze the restrained laminated plates under shear-dominated loading and applied to predict the web local buckling of thin-walled composite beams.

1. Introduction

Laminated composite structures have been increasingly used in aerospace, marine, civil, automobile, and other engineering due to their advantageous properties, such as high strength, lightweight, improved resistance to corrosion and fatigue, and the ability to be tailored to meet specific applications, when compared to traditional materials. On the other hand, due to their high strength and thin-walled nature of most of composite structures, they are susceptible to elastic buckling before reaching to the material strength failure. Thus, accurate methods for the buckling prediction of composite structures are needed.

Buckling of composite laminated plates under in-plane shear has attracted less attention than the ones under compression [1]. Shufrin et al. [2] presented a semi-analytical method for the buckling analysis of symmetrically-laminated rectangular plates under in-plane compression and shear based on the variational principle of total energy and iterative extended Kantorovich method. Zhang and Matthews [3] and Kosteletos [4] examined the buckling behavior of general laminated panels with simply-supported and clamped edges under combined shear and compressive loading based on the Galerkin method. Loughlan [5] used the finite strip method to examine the effect of bending-twisting coupling on the shear buckling behavior of laminated composite constructions. Iyengar and Chakraborty [6] performed the buckling analysis of moderately thick/very thick composite laminated plates under in-plane compressive and shear loading using the finite element method based on a simple higher order shear deformation theory. Jung and Han [7] investigated the buckling response of laminated composite plates and shells under the combined in-plane loading using a modified 8-ANS method based on a modified first-order shear deformation theory.

The panels with either the simply supported or clamped boundary condition are the extreme cases; while in reality, the boundary edges of plates or panels are usually elastically-restrained by the adjacent components (e.g., in the thin-walled structures, depending on which components (webs or flanges) buckle first, the flanges are restrained by the webs, or the webs are restrained by the flanges) and the scenarios between the simply-supported and clamped boundary conditions. The buckling behavior of laminated plates with the edges elastically-restrained against rotation under compression has been extensively investigated by many researchers. Ni et al. [8] presented a buckling analysis for a rectangular laminated composite plate with arbitrary edge supports subjected to the biaxial compression loading based on the higher-order shear deformation theory. They introduced the *pb*-2 Ritz functions which could express an arbitrary edge support condition into the Rayleigh–Ritz method. Mittelstedt [9] investigated the buckling behavior of symmetric rectangular laminated plates using the Ritz method. The considered laminates are subjected to a linearly-distributed in-plane compressive normal load, and they are simply supported at the two loaded edges with one free unloaded plate edge and one simply-supported unloaded edge where the elastic rotational restraints are considered. Qiao and Shan [10,11] used a variational formulation of the

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Ritz method to present the explicit solution in terms of the rotational restraint stiffness for the local buckling behavior of composite plates elastically-restrained along their four edges and subjected to the biaxial compression and uniaxial compression, respectively. Gorman [12] employed the superposition method to obtain the buckling loads for a family of elastically-supported rectangular plates subjected to uniaxial compression. Paik and Thayamballi [13] investigated the buckling strength characteristics of steel plates elastically-restrained at their edges. They derived the characteristic equation for the buckling strength of steel plates which are elastically restrained along either the long or short edges and simply supported along the other two opposite edges. Lai and Xiang [14] presented a discrete singular convolution (DSC) method to solve the buckling problems of rectangular plates with all edges transversely-supported and restrained by the uniform elastic rotational springs. The opposite plate edges are subjected to a linearly-varying uniaxial in-plane loading. Bank and Yin [15] presented solutions and a parametric study for the buckling of rectangular orthotropic plates subjected to uniform uniaxial compression. The plates considered were simply supported on the loaded edges and free and elastically restrained on the other unloaded edge.

Concerning the plates or panels with the elastically-restrained edges subjected to shear loading, Qiao and his coworkers [1] have recently conducted some work on the relatively-long plate (e.g., the plate aspect ratio $\gamma = a/b \geq 5$). Qiao and Huo [1] presented the explicit and approximate closed-form local buckling solution of in-plane shear loaded orthotropic plates with two opposite edges simply-supported and other two opposite edges either both rotationally-restrained or one rotationally-restrained and the other free. Qiao et al. [16] studied two cases of composite plate analyses with two opposite edges simply supported and the other two opposite edges either both rotationally-restrained or one rotationally-restrained and the other free, and they presented the explicit buckling expressions of the plate in generic loading cases of combined linearly-varying axial and in-plane shear loading using the Ritz method. Liu et al. [17] presented the buckling analysis of orthotropic plates with two opposite edges simply-supported and the other two opposite edges rotationally-restrained and under combined uniform in-plane shear and linearly varying axial loads and extended the solution to the web local buckling predictions of FRP structural shapes. Though the above recent studies on the shear buckling of restrained plates have been presented [1], they are very approximate in nature for the sake of deriving the explicit or semi-explicit formulas which are only applicable to the relatively-long plate with the aspect ratio of $\gamma = a/b \geq 5$; thus, they are not applicable to the laminates with the finite length (e.g., square laminates). To the authors' knowledge, there is little literature on the topic of shear buckling of finite length panels with the elastically-restrained edges. Huang and Thambiratnam [18] proposed a procedure incorporating the finite strip method together with the spring systems for treating plates on elastic supports, and the spring systems can simulate different elastic supports, such as the elastic foundation, line and point elastic supports as well as the mixed boundary conditions.

In this paper, the authors present a semi-analytical solution for the finite length composite laminated plates (as shown in Fig. 1) with all the four edges rotationally-restrained and subjected to combined shear and compression using the Galerkin method. The eigenfunctions of simply supported and clamped cases are combined to uniquely satisfy the rotationally-restrained boundary conditions. The present results are compared with those from the numerical finite element analysis. Then, a parametric study is conducted to examine the effect of a range of parameters, i.e., the rotational restraint stiffness, aspect ratio, and orthotropy and anisotropy parameters on the buckling behavior of rotationally-restrained plates under the shear or combined shear and compressive loading action.

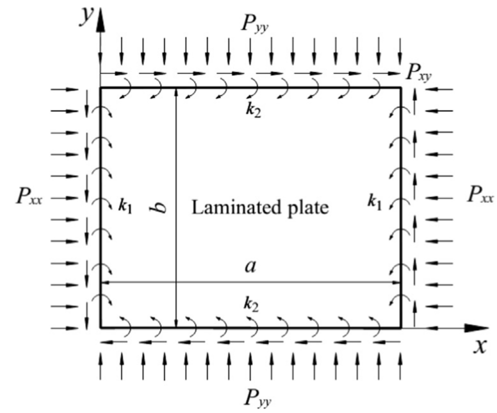

Fig. 1. Geometry of the rotationally-restrained composite laminated plates under shear or combined shear and compression action.

2. Theoretical formulations

2.1. Governing equation

The laminated plate and coordinate systems for the intended study are shown in Fig. 1, and the length and width of the plate are a and b, respectively. The laminated plate is subjected to the in-plane shear P_{xy} or the combined shear P_{xy} and bi-axial compression P_{xx} and P_{yy}; in addition, the plate is elastically restrained along all the four edges with the rotational restraint stiffness k_1 at $x=0$ and a and k_2 at $y=0$ and b. The laminated plate considered is thin (i.e., the plate thickness h is much smaller than the in-plane dimensions of the plate), and its layup is in the symmetric configuration (i.e., the extension-bending stiffness coefficients B_{ij} ($i,j=1,2,6$) are zero), so the classical laminate plate theory is applied. The constitutive relations for a symmetric laminated plate are expressed as:

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{1}$$

where M_{xx}, M_{yy} and M_{xy} are the bending and twisting moments per unit length, w is the transverse deflection of every point (x, y) of the middle surface of the plate, and D_{ij} (i, $j=1$, 2, 6) are the bending stiffness (see [19,20]).

The equilibrium equation of a symmetrically-layered laminate under in-plane loading at buckling is given as [21]:

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial xy} + \frac{\partial^2 M_{yy}}{\partial y^2} + P_{xx}\frac{\partial^2 w}{\partial x^2} + 2P_{xy}\frac{\partial^2 w}{\partial xy} + P_{yy}\frac{\partial^2 w}{\partial y^2} = 0 \tag{2}$$

By substituting Eq. (1) into Eq. (2), the following governing equation is obtained:

$$\frac{\partial^4 W}{\partial \xi^4} + a_1\frac{\partial^4 W}{\partial \xi^3 \partial \eta} + a_2\frac{\partial^4 W}{\partial \xi^2 \partial \eta^2} + a_3\frac{\partial^4 W}{\partial \xi\, \partial \eta^3} + a_4\frac{\partial^4 W}{\partial \eta^4}$$
$$-\lambda a_5\left(N_1\frac{\partial^2 W}{\partial \xi^2} + 2\gamma S\frac{\partial^2 W}{\partial \xi\, \partial \eta} + \gamma^2 N_2\frac{\partial^2 W}{\partial \eta^2}\right) = 0 \tag{3}$$

where the following non-dimensional parameters are considered:

$$W = \frac{w}{h}, \quad \xi = \frac{x}{a}, \quad \eta = \frac{y}{b}, \quad \gamma = \frac{a}{b}$$