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On distortion of symmetric and periodic open-section thin-walled members

Rodrigo Gonçalves [a,*], Dinar Camotim [b]

[a] CEris, ICIST, Departamento de Engenharia Civil, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal
[b] CEris, ICIST DECivil, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

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ABSTRACT

This paper discusses aspects related to the mechanics underlying the distortion of thin-walled members with symmetric and periodic open cross-section, such as those commonly employed in cold-formed steel construction. The Generalised Beam Theory (GBT) framework is employed to determine the cross-section distortional deformation modes and obtain insight into the problem under consideration. Besides reviewing the well known case of reflectional symmetry, the implications of rotational symmetry and periodicity through translation or glide reflection are examined. For each case, computationally efficient procedures to obtain the distortional modes are provided. Several examples are presented throughout the paper, in order to enable a better grasp of the concepts and procedures addressed.

1. Introduction

It is well known that the structural analysis of symmetric systems may be based on substructure simplifications, a procedure which leads to a significant DOF reduction and, perhaps most importantly, helps understanding the structural behaviour of the complete system. The latter aspect is particularly important in the case of thin-walled members, where substructure simplifications may help grasping the mechanics of cross-section in-plane and out-of-plane (warping) deformation. In fact, thin-walled members invariably exhibit cross-section symmetry, a feature that renders them an ideal candidate for this type of approach.

It is also well known that cross-section distortion — a deformation mode type involving cross-section in-plane and out-of-plane (warping) deformation, with displacements of the cross-section fold-lines — plays a major role in the structural behaviour of thin-walled members. For instance, this mode type has been a subject of intensive investigation in the field of cold-formed steel member stability, namely for lipped channel, zed, hat or "rack" sections (see, e.g., [chapter 13 in 1]). In this respect, the Generalised Beam Theory (GBT, see [2,3]) has been established as a very efficient and clarifying tool, due to its capabilities of straightforwardly including/eliminating specific effects. In particular, GBT often leads to analytical or semi-analytical solutions that make it possible to extract unique and in-depth information concerning the member structural behaviour.

In the context of GBT, the classic reflectional symmetry simplification procedure is described in [2] and has been employed in the past in numerical applications (e.g., [4,5]) and also in the derivation of analytical formulae for the distortion of zed, lipped channel and hat sections [2,3,6]. Recently, the rotational symmetry of regular convex polygonal tubes was explored to obtain insight into their first-order, buckling and vibration behaviours [7–10].

This paper explores the implications of cross-section symmetry and periodicity on the characteristics of the distortional deformation modes of thin-walled members with open cross-section, using a GBT-based approach. Besides reviewing the well known case of cross-section reflectional symmetry, the implications of rotational symmetry and periodicity (through either translation or glide reflection along a straight line) are discussed and computationally efficient procedures for computing the distortional modes are provided for each case. The outline of the paper is as follows. Section 2 presents a brief review of the GBT procedure for calculating the distortional deformation modes of open sections (either branched or unbranched). Sections 3 and 4 are devoted to the reflectional and rotational symmetry cases, respectively, and Section 5 focuses on finite and infinite periodic cross-sections. The paper then closes in Section 6, with the concluding remarks.

2. GBT distortional modes for open sections

A notation similar to that of previous papers is followed (e.g., [4,11]), with matrices in boldface uppercase letters and vectors in boldface lowercase letters. Furthermore, (x,y,z) are midsurface

* Corresponding author. Tel.: +351 21 2948580; fax: +351 21 2948398.
E-mail address: rodrigo.goncalves@fct.unl.pt (R. Gonçalves).

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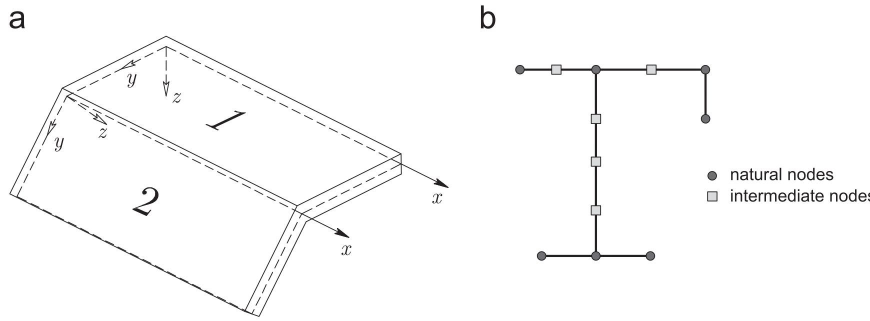

Fig. 1. (a) Thin-walled member local coordinate systems. (b) GBT cross-section discretisation for an arbitrary open cross-section.

local axes at each wall, shown in Fig. 1(a), the comma indicates differentiations (e.g., $f_{,x} = \partial f / \partial x$) and superscripts M and B designate membrane and bending terms, respectively.

The displacement components of the wall mid-surface ($z=0$) are given by

$$u(x,y) = \sum_{k=1}^{D} \overline{u}_k(y)\phi_{k,x}(x), \tag{1}$$

$$v(x,y) = \sum_{k=1}^{D} \overline{v}_k(y)\phi_k(x), \tag{2}$$

$$w(x,y) = \sum_{k=1}^{D} \overline{w}_k(y)\phi_k(x), \tag{3}$$

where D is the number of deformation modes, $\overline{u}_k(y)$, $\overline{v}_k(y)$, $\overline{w}_k(y)$ are the deformation mode displacement components along x, y, z, respectively, and $\phi_k(x)$ are their amplitude functions along the beam length (the problem unknowns). The displacement components for $z \neq 0$ are obtained from the mid-surface ones, using Kirchhoff's thin plate assumption.

The GBT deformation modes are obtained by analysing the cross-section as a plane frame, whose DOFs correspond to the displacements of (see Fig. 1(b)): (i) "natural" nodes, user-independent and automatically located at wall mid-line intersections and free edges, and (ii) "intermediate" nodes, user-dependent and arbitrarily positioned between the natural nodes, in order to achieve a given discretisation level. Since the distortional modes are determined from the displacements of the natural nodes only, no intermediate nodes are considered in this paper.

For thin-walled open sections, following the classic GBT approach [2], it is generally acceptable to assume null membrane shear strains and null membrane transverse extensions, leading to

$$\gamma_{xy}^{M} = 0 \;\Rightarrow\; \overline{u}_{k,y} = -\overline{v}_k, \tag{4}$$

$$\varepsilon_{yy}^{M} = 0 \;\Rightarrow\; \overline{v}_k = \text{constant in each wall}, \tag{5}$$

for each mode $k = 1, \ldots, D$, and therefore $\overline{u}_k$ must be linear in each wall. The $\overline{w}_k$ functions are obtained by analysing the cross-section as a plane frame, subjected to imposed $\overline{v}_k$ displacements, and it follows from Eq. (4) that the deformation modes associated with warping displacements of the natural nodes are univocally defined by the warping functions $\overline{u}_k$. These modes are designated as "Vlasov natural modes" and include the classic "rigid-body" modes (axial extension, major and minor axis bending and torsion) plus distortional modes. For open sections without branches, the number of warping functions equals the number of natural nodes, whereas with branches not all nodes may undergo independent warping displacements, due to the wall connectivity (see, e.g., [12]). A general procedure for identifying the independent nodes is given in [4] and an initial set of Vlasov modes is simply obtained by imposing unit warping displacements at each one, separately, and calculating the associated $\overline{v}_k$ and $\overline{w}_k$ functions.

In order to obtain the distortional deformation modes, it is first necessary to examine the homogeneous form of the GBT equation system, which is given by

$$\mathbf{C}\boldsymbol{\phi}_{,xxxx} - \mathbf{D}\boldsymbol{\phi}_{,xx} + \mathbf{B}\boldsymbol{\phi} = \mathbf{0}, \tag{6}$$

where vector $\boldsymbol{\phi}$ groups the amplitude functions. For the natural Vlasov modes, the so-called GBT modal matrices are symmetric and given by

$$\mathrm{B}_{ik} = \int_S D_f \overline{w}_{i,yy} \overline{w}_{k,yy}\, dy, \tag{7}$$

$$\mathrm{C}_{ik} = \int_S \left(Et\overline{u}_i\overline{u}_k + D_f\overline{w}_i\overline{w}_k \right) dy, \tag{8}$$

$$\mathrm{D}_{ik} = \int_S \left(\frac{Gt^3}{3}\overline{w}_{i,y}\overline{w}_{k,y} - \nu D_f(\overline{w}_{i,yy}\overline{w}_k + \overline{w}_{k,yy}\overline{w}_i) \right) dy, \tag{9}$$

where $D_f = Et^3/(12(1-\nu^2))$, S denotes the cross-section mid-line (along y), t is the wall thickness and the material parameters E, G, ν are Young's modulus, shear modulus and Poisson's ratio, respectively. The distortional modes are the eigenvectors associated with the non-null eigenvalues of

$$(\mathbf{B} - \lambda\mathbf{C})\boldsymbol{v} = \mathbf{0}, \tag{10}$$

where $\mathbf{C}$ concerns membrane and bending warping displacements and is positive definite, whereas $\mathbf{B}$ concerns transverse curvatures and is positive semi-definite with a nullspace associated with the cross-section rigid-body modes.

The eigenvalue problem defined by Eq. (10) constitutes a key step in the calculation of the distortional modes and it is to be expected that (at least) some features of the resulting modes may be predicted by simple inspection of the matrices involved. In particular, properties associated with geometric symmetry/periodicity may be extracted. However, to achieve far-reaching conclusions, it is essential to devise a cross-section discretisation that reflects the geometrical features, i.e., the node numbering must mimic the relevant geometrical features, as discussed in the following sections.

One final word to recall that, concerning the natural Vlasov modes for open sections with n independent natural nodes, the number of distortional deformation modes equals $n-4$.[1]

3. Reflectional symmetry

Open cross-sections exhibiting symmetry with respect to reflection are common, e.g., standard I and lipped channel sections. For this well known symmetry type, the deformation mode shapes can be

[1] For branched sections, $n = n_{nodes} - N_w + 2$, where n_{nodes} is the number of natural nodes and N_w is the number of free end nodes [11].