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Stability of cantilever walls of steel thin-walled bars with open cross-section

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A B S T R A C T

In thin-walled bars with open cross-section, built from flat walls, cases are found in which, the cantilever wall, elastically restrained in the web, is compressed, and at the same time, longitudinal variation in stresses along its length occurs. Such a wall can be analyzed as a cantilever thin plate, elastically restrained against rotation on the longitudinal supported edge. The paper presents the results of investigations into the stability of elastically restrained cantilever plates for varied stress intensity in the longitudinal direction. Graphs of plate buckling coefficients (k) were determined and approximation formulas were derived for technically crucial stress distributions and different degrees of the plate edge elastic restraint. The impact of longitudinal stress variation and of the degree of elastic restraint on the plate buckling modes was analyzed.

1. Introduction

Modern design technologies for thin-walled members, e.g. those presented in [1], are intended to account for a number of parameters that heighten the accuracy of representing the real service conditions of a structural member in the computational model. The relevant parameters include the following: 1) conditions of elastic restraint of walls (component plates) in the thin-walled bar segment, and 2) longitudinal stress distribution depending on the way the member is loaded. The thin-walled bar segment was defined in [2] as a bar length between transverse stiffenings (ribs, diaphragms, etc.) which assure a stiff cross-section contour (Fig. 1).

In thin-walled bars with open cross-section, flat cantilever walls can occur, the loading of which varies along the length, and which are elastically restrained in the so-called internal wall (e.g. in the web) (see Fig. 1). It should be emphasized that as regards local stability, cantilever walls are characterized by much lower resistance to compressive stresses when compared with internal walls.

Correctly determining critical stresses of local buckling for walls (component plates) of the thin-walled bar, supported and loaded as indicated above, specifies the range of the pre-buckling behavior of the member and may be used to more accurately estimate the limit load-carrying capacity on the basis of the effective width method.

The monograph on the stability of thin plates [3] provides graphs and approximation formulas for plate buckling coefficients (k) for axially compressed cantilever plates and internal plates (elastically restrained on one or two longitudinal edges) at constant stress intensity over their lengths. While using those formulas, however, it should be remembered that they describe the first half-wavelength of the plate buckling. For longer plates (e.g. those constituting the component walls of thin-walled bars), the buckling mode is characterized by many half-waves formed over their lengths. To determine coefficients k of successive buckling half-waves, it is necessary to determine subsequent repetitions of the graph, e.g. in accordance with the procedure presented in [4]. The means most frequently used in practice, however, is to assume that $k = k_{min}$ for the buckling first half-wavelength, which, at the same time, corresponds to the coefficient value for the indefinitely long plate.

Yu and Schafer [5] analyzed the impact of longitudinal stress variation on the stability of axially compressed cantilever and internal plates. Linear distribution of normal stresses over the plate length was obtained by introducing shear stresses. An approximate formula for computing the buckling coefficient (k) of cantilever and internal plates for limit boundary conditions (pin or build-in) on longitudinal supported edges in the load range $-1 \le r \le 1$ where $r = \sigma_1/\sigma_0$ (see Fig. 1) was proposed. In study [6], the impact of longitudinal stress variation on the limit load-carrying capacity of axially compressed plates constituting separate (i.e. simply supported) component walls of the thin-walled members with an open cross section was examined. The results obtained with the finite element method were compared with the effective width method in accordance with Winter's formula [7].

Another means of obtaining stress variation along the length of axially compressed internal plate was employed by Kowal in studies [8,9]. The plate was loaded with uniformly distributed forces on

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Nomenclature

- b_s, t_s width, thickness of a plate (wall s)
- C_θ rotational spring stiffness of the plate edge equal to the bending moment resulting from the rotation by a unit angle
- D plate (wall) flexural rigidity
- E Young's modulus of elasticity
- f_{ip} dimensionless, free parameters of the deflection function
- h_w web height in flange midlines
- i_o number of half-wavelengths of the sine function in the plate length direction
- k plate buckling coefficient
- L_s work done by external forces
- l_s plate (wall s) length, length of the thin-walled bar segment
- l_{cr} buckling length of the critical half-wave
- m parameter of the longitudinal stress distribution in accordance with (9)
- p_o polynomial degree
- U_s total potential energy of the system
- V_{s1} elastic strain energy of the plate bending
- V_{s2} energy of the elastic restraint against rotation of the longitudinal edge ($y_s=0$),
- w_s function of the plate (wall s) deflection
- x_s, y_s, z_s Cartesian coordinates of the plate (wall s)
- X longitudinal body forces
- $\beta(x_s)$ function of normal stress distribution in the direction of the plate (wall s) length
- $\gamma_s = l_s/b_s$ geometric ratios of the plate (wall s)
- ν Poisson's ratio
- σ_x, τ_{xy} membrane stresses
- σ_{cr} critical stress of local buckling
- σ_E Euler's stress $= \pi^2 t_s^2 E/(12b_s^2(1-\nu^2))$
- σ_0, σ_1 edge normal stresses (positive, if compressive)
- ε coefficient of elastic restraint
- κ index of fixity

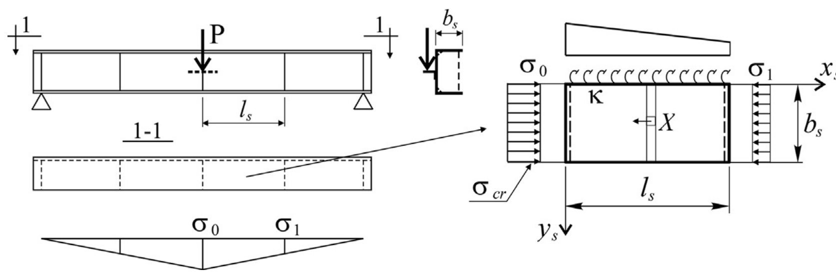

Fig. 1. Cantilever plate separated out of a thin-walled bar segment at longitudinal stress variation.

transverse edges and with longitudinal body forces applied to its middle plane. The problem was solved with the Galerkin method by calculating buckling coefficients (k) of plates with different side length ratios and various options as regards the longitudinal stress distribution.

Szychowski [2] presented the results of investigations into the stability of axially and eccentrically compressed cantilever plates at longitudinal stress variation for boundary conditions of the support of one longitudinal edge (simple support or build-in). Stress variation over the plate length was obtained by introducing longitudinal body forces. Formulas for the work done by external forces at the load inducing longitudinal stress distribution were derived in accordance with a linear function and with 2nd degree parabola. Graphs of buckling coefficients (k) were determined for variously loaded cantilever plates for the parameter $m=1-r$ in the range $0\le m\le 1$. It was demonstrated that for a built-in cantilever plate, at longitudinal stress variation, local buckling “half-waves”, different in length and of varied (e.g. decreasing) amplitude, can be formed.

To obtain a technical solution to many problems of local buckling and to compute the limit load-carrying capacity (estimated on the basis of the effective width method) of thin-walled bars with open cross-section, we need solutions, which are not available, to the critical state of cantilever walls (plates) elastically restrained on one side when loads inducing longitudinal stress variation occur. Fig. 2. shows the graphs of coefficients k for linear stress distribution (for $r=0$), which can be computed using the approximation formulas used in studies [5,6] (dashed lines). In real thin-walled bars, cantilever walls are generally elastically restrained in internal walls (e.g. in webs), which could be utilized in the computational model. In this case, coefficient k for wall *b1* of the thin-walled bar is contained in the range shown in the brace in Fig.2.

The present study intends to determine critical stress and mode of local buckling of cantilever plates elastically restrained on one-side (e.g. in the web of the thin-walled bar) and axially compressed at longitudinal stress variation. The plate transverse edges were assumed to be simply supported (see Fig. 1). Also, approximation formulas for plate buckling coefficients, which facilitate computations of the bearing capacity of thin-walled bars with open cross-sections, were developed.

2. Degree of elastic restraint

In technical literature on the subject, two ways of the description of the degree of elastic restraint against rotation of the supported plate edge are employed. Study [3] provides a dimensionless coefficient of elastic restraint of the plate edge in the following form:

$$\varepsilon = C_\theta \cdot b_s/D \quad (1)$$

In accordance with formula (1), the coefficient of elastic restraint varies from $\varepsilon=0$ for simple support, to $\varepsilon=\infty$ for a built-in edge.

In study [10], however, the formula for the index of fixity of the plate edge has the following form:

$$\kappa = \left(1+\frac{2D}{b_s C_\theta}\right)^{-1} \quad (2)$$