Thin-Walled Structures 94 (2015) 457–465

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Buckling of cracked cylindrical panels under axially compressive and tensile loads

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ARTICLE INFO

Article history:
Received 8 March 2015
Received in revised form
28 April 2015
Accepted 14 May 2015
Available online 3 June 2015

Keywords:
Buckling
Tensile and compressive loads
Panel
Crack

ABSTRACT

Buckling collapse of thin-walled structures under compression is one of the critical type of failures. These structures also may be locally buckled under tensile load with special conditions. In this paper, compressive and tensile buckling of thin cracked cylindrical panels are investigated. Effects of various factors such as the position, length and direction of crack, length, width and thickness of the panel and boundary conditions on the compressive and tensile buckling loads are determined. The results show that the dimensions of panels have lowest while characteristics of crack and closing to edges have greatest effects on the buckling load.

1. Introduction

Today, use of thin walled structures in different branches of engineering is very common. They are used, for instance in construction of tanks, marine structures and aerospace components. In these structures, different types of defects can be induced due to the result of some destructive processes such as corrosion or fatigue loadings. Cracks in the structures and mechanical components can reduce significantly the ultimate bearing capacity and strength. Since the design process requires to assess the safety, many researchers in recent years have tried to describe the various factors that may affect the buckling under compressive or tensile global loads. In this regard, experimental and analytical methods and numerical modeling are used.

Carlson et al. [1] performed an experimental study on the buckling of thin cracked plates under tensile loads and presented an empirical formula for tensile critical buckling stress of cracked plates. Their empirical formula relates the buckling stress to the Young's modulus and square of thickness to crack length ratio. Zielsdorff and Carlson [2] studied on the buckling in cracked thin sheets and out of plane deflection parameters around the crack. Markstrom and Strorakers [3] used the finite element method based on the linear bifurcation theory to determine the buckling load of the cracked elastic plates under uniaxial tensile loads. Sih and Lee [4] investigated numerically the types of buckling modes in a plate with vertical or horizontal central crack under tension and compression. They found that the critical buckling loads decrease by increasing crack length for both loading cases. Shaw and Huang [5] used the numerical method based on the Von Karman's linearized theory for studying the buckling of tensioned cracked plates. They determined the effects of initial imperfections, crack size, boundary conditions, Poisson's ratio and biaxial loading. Buckling and postbuckling of cracked plates loaded in tension were analyzed by using the finite element method [6]. Results show that the local buckling around the crack increases the stress intensity factor due to the redistribution of the stress fields.

Barut et al. [7] numerically studied the local tensile buckling and fracture response of a thin flat composite plate with an inclined interior crack. The results indicate that the buckling load increases and the energy release rate decreases as the crack orientation changes from normal to parallel direction with respect to tensile load. Buckling of cylindrical shells and panels with through cracks has been studied numerically with finite element models consisting of quadrilateral isoparametric refined shell elements around the crack tip [8,9]. Under tensile loads, the curvature of panel results in a higher buckling load and considerable out of plane displacements. This normal displacements remain nearly constant and localize around the crack by increasing the curvature.

Guz and Dyshel [10] studied the buckling or crack growing of thin plates with an edge crack with different boundary conditions under tensile load. Brighenti [11,12] analyzed numerically the

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http://dx.doi.org/10.1016/j.tws.2015.05.011
0263-8231/

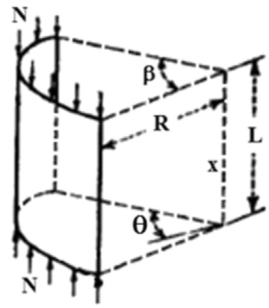

Fig. 1. Cylindrical panel under axial compression.

effects of various geometrical and mechanical properties and boundary conditions on the buckling of cracked plates under tensile or compressive loads. He proposed an approximate analytical model for cracked plates in tension by considering the localized buckling phenomena and supposing an embedded compressed portion of the tensioned plate. The obtained results by Brighenti [13] show that the buckling of cracked plates under compressive stress heavily depends on the boundary conditions, while they are almost independent for tension cases. In this case the crack always reduces the buckling load with respect to the undamaged one. Paik et al. [14] carried out an experimental and numerical study on the ultimate strength of cracked plate subjected to axial compressive or tensile loads. They used a simple relation to predict the ultimate strength of the cracked plate subjected to tensile loads based on the reduced cracked cross-sectional area. Datta and Biswas [15] reviewed the tensile buckling of aerospace structural elements with attention to the local buckling and parametric excitation due to periodic loading on the plates and curved panels. Jahromi and Vaziri [16] numerically studied the compressive buckling of singly and doubly cracked cylindrical shells. The buckling load of panels with single longitudinal crack may be affected most detrimental with respect to angled or circumferential cracks. The local buckling shape mainly depends on the crack angle and is insensitive to the crack length and shell thickness. For shells with multiple cracks, the buckling load and shape are influenced not only by the buckling behavior of each individual crack but also by the interaction between the cracks. Kim et al. [17] studied numerically the effect of using an elastic liner on the stability of a cracked cylindrical shell. Their results show that the strength of cracked shell and buckling mode can be largely influenced by thickness and relative stiffness of the liner layer.

As can be seen, most of studies were performed on the plates and cylindrical shells with axial or transverse cracks. In this study, the buckling of curved plates (panels) is investigated. Effects of various parameters such as crack length and orientation, thickness and radius of panel and boundary conditions on the buckling under compressive and tensile axial loads have been determined.

2. The compressive buckling load of perfect cylindrical panel

An open cylindrical shell where denoted as panel is depicted in Fig. 1. It has radius R, axial length L, thickness t and central angle β. A panel as a curved plate is two dimensional element and its behavior can be characterized completely by its middle surface. Thus, a 2-D coordinate system attached to the middle surface can be used to describe its behavior and specially the buckled shape. For the circular cylindrical panels, it consists of longitudinal coordinate, x and circumferential coordinate θ or the arc length $y = R\theta$, as shown in Fig. 1.

When compressive load applied over a thin panel, for a certain load value, the unstable buckling occurs. Buckling of panels are similar to plates with some differences due to curvature effects.

If u, v and w are respectively the axial, circumferential and radial displacements of the panel, equilibrium equations can be written as follows when only resultant force in the x direction, $N_x = -N$, is applied [10]:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1+\nu}{2R}\frac{\partial^2 v}{\partial x \partial \theta} - \frac{\nu}{R}\frac{\partial w}{\partial x} + \frac{1-\nu}{2}\frac{\partial^2 u}{R^2 \partial \theta^2} = 0$$

$$\frac{1+\nu}{2}\frac{\partial^2 u}{\partial x \partial \theta} + \frac{R(1-\nu)}{2}\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{R \partial \theta^2} - \frac{1}{R}\frac{\partial w}{\partial \theta}$$

$$+ \alpha\left[\frac{\partial^2 v}{R\partial\theta^2} + \frac{1}{R}\frac{\partial^2 w}{\partial \theta^2} + R\frac{\partial^2 w}{\partial x^2 \partial \theta} + R(1-\nu)\frac{\partial^2 v}{\partial x^2}\right] - R\phi\frac{\partial^2 v}{\partial x^2} = 0$$

$$-R\phi\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial u}{\partial x} + \frac{1}{R}\frac{\partial v}{\partial \theta} - \frac{w}{R} - \alpha\left[\frac{\partial^2 v}{R\partial\theta^2} + (2-\nu)R\frac{\partial^2 v}{\partial x^2 \partial \theta} + R^2\frac{\partial^4 w}{\partial x^4}\right.$$

$$\left. + \frac{1}{R}\frac{\partial^4 w}{\partial \theta^4} + 2R\frac{\partial^4 w}{\partial x^2 \partial \theta^2}\right] = 0 \tag{1}$$

where $\alpha = t^2/12R^2$ and $\phi = N(1-\nu^2)/Et$.

By assuming the simply supported edges scheme on all sides of the panel, the axial, circumferential and radial displacements can be expressed in terms of the trigonometric functions as follows:

$$u = \sum_{m,n} A_{mn} \sin\frac{n\pi\theta}{\beta}\cos\frac{m\pi x}{L}$$

$$v = \sum_{m,n} B_{mn} \cos\frac{n\pi\theta}{\beta}\sin\frac{m\pi x}{L}$$

$$w = \sum_{m,n} C_{mn} \sin\frac{n\pi\theta}{\beta}\sin\frac{m\pi x}{L} \tag{2}$$

By substituting Eq. (2) in (1) and assuming $\lambda = m\pi R/L$, we have

$$A_{mn}\left(\lambda^2 + \frac{1-\nu}{2}(\frac{n\pi}{\beta})^2\right) + B_{mn}\frac{n\pi\lambda}{2\beta}(1+\nu) + C_{mn}\nu\lambda = 0$$

$$A_{mn}\frac{n\pi\lambda}{2\beta}(1+\nu) + B_{mn}\left[\lambda^2((1-\nu)(0.5+\alpha)) - \phi\right.$$

$$\left. + (\frac{n\pi}{\beta})^2(1+\alpha)\right] + C_{mn}\frac{n\pi}{\beta}\left[1 + \alpha\left(\lambda^2 + (\frac{n\pi}{\beta})^2\right)\right] = 0$$

$$A_{mn}\nu\lambda + B_{mn}\frac{n\pi}{\beta}\left\{1 + \alpha\left[(\frac{n\pi}{\beta})^2 + (2-\nu)\lambda^2\right]\right\}$$

$$+ C_{mn}\left[1 - \lambda^2\phi + \alpha\left(\lambda^2 + (\frac{n\pi}{\beta})^2\right)^2\right] = 0 \tag{3}$$

In order to get the nontrivial solution for displacements, the determinant of coefficients matrix must be zero. For thin panels, the values of α and ϕ are well below the unit value, so their square can be omitted. The nontrivial solution of (3) gives