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# Thermal wrinkling of thin membranes using a Fourier-related double scale approach

Kodjo Attipou [a,b,*], Heng Hu [c], Foudil Mohri [b], Michel Potier-Ferry [b], Salim Belouettar [a]

[a] Centre de Recherche Public Henri Tudor, 29, avenue John F. Kennedy, L-1855 Luxembourg-Kirchberg, Luxembourg
[b] Université de Lorraine, Laboratoire d'Étude des Microstructures et de Mécanique des Matériaux (LEM3), UMR CNRS 7239, Ile du Saulcy, F-57045 Metz Cedex 01, France
[c] School of Civil Engineering, Wuhan University, 8 South Road of East Lake, 430072 Wuhan, PR China

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ABSTRACT

The thermal wrinkling behavior of thin membranes is investigated in this paper. Wrinkles often occur at multiple length scales where induced compressive stresses are located during thermal loading. In the present study, the method of double scale Fourier series is used to deduce the macroscopic membrane wrinkling equations. The obtained equations account for the global and local wrinkling modes. Numerical examples are conducted to assess the validity of the approach developed. The present membrane's model needs only few degrees of freedom to recover more accurately the post-buckling equilibrium curves and the wrinkling shapes. Different parameters such as membrane's aspect ratio, wave number, and pre-stressed membranes are discussed from a numerical point of view and the properties of the wrinkles (critical load, wavelength, size, and location) are presented.



## 1. Introduction

Membrane structures are widely used in many engineering fields such as aeronautic, space, automotive and civil construction, as well as skydiving. For instance, gossamer structures are used in the space industry for satellites and spacecrafts. In civil construction, thin membranes such as PVC or aluminum are used in roof covering of large space area as in sport area and exhibition halls. Aeronautical industry uses thin polymer membranes to strengthen airplane wings while keeping them more lighter. Thin membranes are often subjected to stresses and strains due to their applications. For very thin membranes, it can lead to an out-of-plane deformation of the structures because of no flexural stiffness. In some applications, the membranes can be exposed to high temperature changes coupled with mechanical tensile and/or compressive stresses (satellites, PVC manufacturing, etc.). In relatively large thin membranes, thermal buckling occurs locally and repetitively and results into wrinkles. The buckling or wrinkling phenomena are instabilities that can damage the membrane structures when they are not controlled. Controlling the buckling phenomenon means assessing its occurrence conditions in order to predict and simulate it. Buckling and wrinkling are caused by compressive stresses. They are all boundary conditions and imperfections dependent. Buckling modes involve length scale in the same order as the size of the structure. Wrinkling is observed especially in membrane structures with a modal length scale much smaller than the membrane's dimensions. This last instability is a complex non-linear phenomenon and its modeling is an interesting and up-to-date research topic that deserves attention. The scope of this study is the wrinkling of thin membranes under thermal loading.

The buckling and wrinkling phenomena in membrane have been studied in the literature. Among these papers, many are focused on the wrinkling of membrane caused by residual stresses. In [1], Cuong et al. have applied thermal stresses on a thin membrane to simulate the residual stresses that appear during the rolling process of a sheet in manufacturing. Residual stresses can also be induced by the growth of living tissues such as mucosa [2]. The most common case where residual stresses are induced is the case of thin membranes subjected to mechanical loads. For instance, Kim et al. [3] have conducted a modified Yoshida buckling test to investigate the wrinkling initiation caused by the mechanical properties as well as the loading case.

There are several techniques to study the wrinkling phenomena in thin membranes: experimental, analytical and numerical. Benchmarks tests have been developed by many authors in the literature. Blandino et al. [4] have proven through experimental

* Corresponding author at: Université de Lorraine, Laboratoire d'Étude des Microstructures et de Mécanique des Matériaux (LEM3), UMR CNRS 7239, Ile du Saulcy, F-57045 Metz Cedex 01, France.
*E-mail address:* kodjo.attipou@yahoo.fr (K. Attipou).

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results that the number of wrinkles increases with the mechanical loads in isothermal conditions. Moreover, the wrinkles amplitude increases with the thermal loading. Besides benchmarks tests, analytical and/or numerical studies have also been conducted for comparison purpose [5,6]. The aim is to develop reliable methods to solve the wrinkling problems. These methods of resolution can be sort into two main groups: the full shell methods and the reduced methods. Full shell methods involve classical equations that are resolved via finite element analysis using shell elements [7]. Such microscopic models are simply based on elastic shell theory (see for instance [8–12]). The cons of the direct methods is that they required many degrees of freedom, then the computation cost becomes important. This is the reason why some alternative methods as the reduced methods are interesting where it applies. Some reduced methods are based on the modification of the constitutive equations of the wrinkling problems. It follows a simplification of the general equations and then a simplified model is built. In 1987, Roddeman et al. [13,14] have developed a wrinkling model for thin membranes under shear load by using a weighted residual method. Roddeman's model was just limited to the determination of wrinkled regions. No information on the wrinkles characteristics (size, wavelength) are available. Chiu et al. [15] have modified Roddeman's model to account for the thermal effect in thermo-mechanically loaded polymer membranes.

Recently, a new reduced method was introduced by Damil et Potier-Ferry [16,17] to analyze the wrinkling phenomenon in one-dimensional beam model. This method is based on a Fourier-related multiscale approach and it is able to couple the macroscopic response of the structure with the characteristics of the wrinkles. With this approach, it is possible to model the wrinkling behavior of a structure around the bifurcation point and away from where former techniques such as Landau–Ginzburg are no longer valid. More precisely the Landau–Ginzburg approach relies on two asymptotic assumptions: closeness to the bifurcation point and separation of scales. The method described in [16,17] is only based on the separation of scales and a truncation of higher Fourier modes and thus it can remain valid away from the bifurcation, what is confirmed by numerical tests in the literature. The other advantage of the Fourier method is that it requires only few degrees of freedom to recover the wrinkling behavior. Of course the advantages of such a reduced-order model are always counterbalanced by some more or less restrictive assumptions, the main one being here the need of choosing a priori the wavenumber of the wrinkling mode. Applications of Fourier method to sandwich structures were conducted by Liu et al. [18] and Yu et al. [19]. Our present study is based on the work of Damil et al. in [20,21] where the Fourier method has been successfully extended to two-dimensional membrane models. In our work, the thermal effect in thin membrane models is investigated. Thermal stresses are introduced in the classical equations within the framework of the elastic shell theory. The nonlinear problem is solved using an Asymptotic Numerical Method described in [22] for efficiency and reliability purpose.

This paper is divided into three main sections. In Section 2, the equilibrium equations are developed. Classical membrane model equations are recalled within the approximations of von Kàrmàn using the classical plate theory of Love–Kirchhoff. Then macroscopic membrane model's equations are deduced using the Fourier series. The generalized Duhamel–Neumann form of the Hooke's law is considered to account for the thermal stresses. In Section 3, an analytical study is performed to investigate the wrinkling critical load and wavenumber in some specific boundary and load conditions. Section 4 is dedicated to numerical examples where a validation test is performed and different parameters are studied to assess the reliability and efficiency of the Fourier series technique developed. The results of the examples come from the implementation of the new membrane model equations in MATLAB code and the simulation of the classical membrane model in the finite element software ABAQUS for comparison.

## 2. Nonlinear equilibrium equations

### 2.1. Full model equations

The main objective of this paper is to develop a nonlinear macroscopic model of thin membranes subjected to thermo-mechanical loads. In this framework, the related equations of the macroscopic models are derived from the Love–Kirchhoff classical plate theory within von Kàrmàn approximations. An elastic isotropic membrane is considered as the reference model. The generalized Duhamel–Neumann form of the Hooke's law is applied to account for the thermal stresses. One recovers the well-known governing equations of the full membrane model as following:

$$\text{div } \boldsymbol{N} = 0 \tag{1a}$$

$$\boldsymbol{N} = \mathbf{L}^m \cdot (\boldsymbol{\gamma} - \boldsymbol{\alpha}\Delta T) \tag{1b}$$

$$2\boldsymbol{\gamma} = \nabla\mathbf{u} + {}^t\nabla\mathbf{u} + \nabla w \otimes \nabla w \tag{1c}$$

$$\mathbf{D}\nabla^4 w - div(\boldsymbol{N} \cdot \nabla w) = 0 \tag{1d}$$

where $\mathbf{u} = (u, v) \in \mathbb{R}^2$ is the in-plane displacement of the microscopic fields, $w$ the deflection, $\boldsymbol{N} \rightarrow {}^t(N_x\ N_y\ N_{xy})$ and $\boldsymbol{\gamma} \rightarrow {}^t(\gamma_x\ \gamma_y\ 2\gamma_{xy})$ are the membrane stresses and strains. The membrane and bending elasticity tensors are represented respectively by the matrices $\mathbf{L}^m$ and $\mathbf{D}$ in case of a single-layered membrane and its mid-plane set as reference of the displacement components:

$$\mathbf{L}^m = \frac{Et}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{2}$$

$$\mathbf{D} = \frac{Et^3}{12(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{3}$$

with $t$ being the membrane thickness, $E$ the Young's modulus and $\nu$ the Poisson ratio. The thermal properties of the membrane are represented by the expansion coefficient $\boldsymbol{\alpha} \rightarrow {}^t(\alpha_x\ \alpha_y\ \alpha_{xy})$ and the temperature variation $\Delta T$. The material constants ($E$, $\nu$, $\alpha$) are assumed to be independent towards the temperature change.

### 2.2. A simplified macroscopic model

The method of Fourier series consists in replacing the unknowns of the nonlinear full model (Eq. (1)) by the Fourier series which have slowly variable coefficients. Nonlinear macroscopic equations whose unknowns are the Fourier coefficients are recovered. The discussion in this paper is devoted to two-dimensional structures. The reader should refer to the literature [16, 17] for the one-dimensional development. First in our model, the wrinkles patterns are assumed to be in one single direction (the axial $x$ direction) and the wavenumber is assumed to be known. These assumptions are restrictive and limited to simplest case of membrane wrinkling and should be removed in future works when dealing with shear membrane wrinkling. Hence, to be more realistic, a discussion is first made around a multiscale approach with a given wavelength and direction of wrinkles. The local problem unknowns $U$ are sought in Fourier series terms in