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Free vibration analysis of stringer stiffened general shells of revolution using a meridional finite strip method

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ABSTRACT

A deep doubly curved shell element is developed for free vibration analysis of general shells of revolution. The mid-surface of the shell may have an arbitrary shape as well as a variable thickness, and the shell can be closed circumferentially or not. For both the circumferential and meridian directions of the shell element, Lagrange polynomials are used to interpolate the displacement variables. Stringer stiffeners are modeled as discrete curved beams to investigate the free vibration of stiffened shells. The frequencies are compared with the published data, and new examples of axisymmetric shells with positive and negative Gaussian curvature are presented.

1. Introduction

Due to the curvature of their middle surface, shell structures take advantage of membrane as well as bending stiffness to effectively sustain the applied loads, which makes them very stiff for both the in-plane and transverse loads. Among various types of shells, those with axisymmetric structures have extensive uses in many fields of engineering, especially civil, mechanical, architectural and aeronautical engineering. For better understanding the dynamic behavior of structures, many research works have focused on the free vibration characteristics of shells. Different shell theories have been developed during the last decades, and the vibration analysis of shells of revolution has included various types of axisymmetric shells and different methods for solving their governing equations. A comprehensive survey of developments in the field of shell dynamics from 1989 to 2009 may be found in the papers by Qatu et al. [1–3], which review the research on the homogeneous and composite shells and report a classification of studies based on different aspects including shell theory, geometry, method of analysis and complicating effects.

Exact solution for the governing differential equations of shells is possible only for special cases such as cylindrical shells with shear diaphragm supports, and for the most problems, the use of numerical methods is inevitable. The most popular numerical methods that have been employed by researchers to analysis shell structures include finite element method, Ritz method, Galerkin method, boundary element method and generalized differential quadrature method, among which the maximum contribution is devoted to finite element method due to its flexibility in dealing with various boundary conditions and complex geometries. During last four decades, development and utilization of shell finite element have grown rapidly and numerous shell elements have been proposed thus far. Ahmad et al. [4] developed a general curved thick shell element from a three dimensional element. The element displacement field is expressed by shape functions in terms of degrees of freedom including three displacements and two rotations per node on the mid-surface. The so-called degenerated element has the debility that locks when applied to the analysis of thin shells. Afterwards, a great deal of improvements has been implemented by researchers to overcome this debility by applying different techniques such as reduced and selective integration [5–8], enhanced interpolation of transverse shear and membrane strains [9], discrete Kirchhoff constraints [10], construction of a stabilization matrix [11] and employing consistent shell element [12]. Saetta and Vitaliani [13] presented a general curved shell element based on Mindlin–Reissner's theory as an alternative one to the degenerated shell theory. Besides these general elements, various curved shell elements have been developed based on different shell theories for the analysis of thin, thick, shallow and deep shells with specified geometries.

A shell of revolution is a type of shell which is generated by rotating a plane curve around an axis; thus, the geometric characteristics of the generated shell are constant circumferentially. This fact can simplify the analysis of such shells, in which, by choosing one strip as an element in the meridian direction of the

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shell, and computing its stiffness and mass matrixes, one may proceed to solve the problem. Some researchers have used this feature to analyze the vibration of axisymmetric shells. In this regard, Jiang and Olson [14] developed a finite strip formulation in the longitudinal direction of thin cylindrical shells to study their nonlinear dynamic behavior. In longitudinal direction, the singly curved strip utilizes trigonometric and hyperbolic shape functions to satisfy essential boundary conditions. Stringers are also modeled as beam elements attached to the nodal lines of strips. Mohd and Dawe [15] developed a finite strip method for free vibration analysis of prismatic shell structures, made up of circularly curved and flat plates with diaphragm end supports. In order to reduce the number of total degrees of freedom, a substructuring technique was employed to create singly curved superstrips based on both thin and first order shear deformation shell theory. The nonlinear eigenvalue problem was solved using a Strum sequence method. A similar substructuring approach was employed by Tan [16] for predicting the natural frequencies of shells of revolution with arbitrary shape of meridian. Spline functions were used to represent the displacements along the meridian and examples of cylindrical, spherical and hyperboloidal shells were investigated.

Shell structures are often reinforced with stiffeners to achieve a high strength-to-weight ratio. The most frequently used stiffeners in axisymmetic shells are the meridional and the circumferential stiffeners, which are called stringers and rings, respectively. Most of the works on the vibration of stiffened shells have focused on cylindrical and conical shells and only a few papers exist on stiffened doubly curved shells. A popular method for incorporating the effects of stiffeners in the analysis is to smear the mass and stiffness of stiffeners over the shell surface and model the stiffened structures as an orthotropic shell. This technique is used by many researchers for the vibration analysis of stiffened plate and shell structures. Bich et al. [17] studied geometrically nonlinear dynamic response and vibration of imperfect stiffened simply supported doubly curved shallow shells made of functionally graded materials using a smeared stiffeners technique. Duc [18] presented nonlinear dynamic response of functionally graded double curved simply supported thin shallow shells resting on elastic foundations and being subjected to axial compressive load and transverse load. The contribution of stiffeners was accounted by a smeared stiffeners method. Wattanasakulpong and Chaikittiratana [19] used Navier solution method to investigate the free vibration of stiffened functionally graded doubly curved shallow shells with simply supported boundary condition and under thermal environment. The effects of the flexural and extensional stiffness of stiffeners were assumed to be smeared over the shell surface. However, the smeared stiffeners technique is not very accurate and cannot represent the actual behavior of stiffened structure when the stiffeners are not identical and equally and closely spaced so that the wavelength of vibration is smaller than the stiffener spacing.

To overcome this limitation, the stiffeners should be considered as discrete beam or shell elements, depending on their dimensions. Bhimaraddi et al. [20] used a shell finite element model in conjunction with a constant curvature curved beam element for static and free vibration analysis of laminated stiffened shells of revolution and presented examples of stiffened cylindrical and conical shell panels. Shen and Chuang [21] analyzed the dynamic response of stiffened shallow shells using the spline Gauss collocation method. A curved triangular shallow shell finite element in conjunction with a curved beam stiffener element was employed by Sinha and Mukhopadhyay [22] to find the natural frequencies of stiffened plates and shell panels. The stiffeners would have arbitrary positions within the shell elements without having shared nodal lines. Furthermore, they used the same shallow shell finite element for static, free vibration and transient dynamic response analysis of stiffened shells, where the thickness of the basic shell element and stiffener could vary quadratically [23]. Goswami and Mukhopadhyay [24] developed a quadratic shear deformable shell element for free vibration analysis of composite stiffened doubly curved shell panels. The formulation had the advantage of arbitrary locating the stiffeners inside the shell element. Prusty and Satsangi [25] studied the transient dynamic response of composite stiffened plates and shells using a curved quadratic shell element and a curved beam element. The stiffened shell formulation was based on compatibility of displacements at the shell-stiffener interface. Nayak and Bandyopadhyay [26,27] investigated the free vibration characteristics of the isotropic doubly curved stiffened panels using a thin shallow shell element and a curved beam element. Samanta and Mukhopadhyay [28] developed a flat triangle stiffened shell element to predict the natural frequencies of shallow and deep stiffened shells. They presented examples of stiffened cylindrical and spherical shell panels and closed cylinder shells. Sahoo and Chakravorty [29] investigated the free vibration of stiffened composite hypar shells by the finite element method, and proposed some guidelines for designing optimized stiffened shells. Prusty [30] analyzed the free vibration and buckling of laminated composite spherical and cylindrical shells and performed parametric studies of various open and closed section stiffeners using the finite element method. Using the Ritz method, Shi et al. [31] reported free vibration analysis of curvilinearly stiffened shallow shells based on the first-order shear deformation shell theory and a curved beam theory.

It is observed from the literature that most of previous studies on the vibration of stiffened doubly curved shells have dealt with shallow shell panels while, study on vibration of stiffened deep axisymmetric shells with arbitrary meridian has not received worthy attention. In the present study, a deep doubly curved shell element is developed in the meridian direction of shells of revolution for determining the free vibration frequencies of such shells based on the Reissner–Mindlin thick shell theory. Material behavior is assumed to be homogeneous and isotropic. The mid-surface of the shell may have an arbitrary shape as well as a variable thickness. The effect of stringer stiffeners is also considered by modeling them as deep curved beam elements having the same displacement functions as the shell. The present results, which include the effects of shear deformations and rotary inertia, are compared with the existing published values, and also new examples of stiffened and un-stiffened shells are presented.

2. Shell theory

2.1. Geometry

A shell structure can be identified by the geometry of its middle surface and its thickness at each point. The mid-surface of an axisymmetric shell is developed by rotating a curve called meridian around an axis called rotation axis. Fig. 1 shows the cross section of a shell of revolution with positive Gaussian curvature from point A to B and negative Gaussian curvature from point B to C. Thickness of shell (h) can vary along the meridional direction. In the figure, y is the axis of revolution, r is the distance of mid-surface from the y axis, and φ is the angle between the line normal to mid-surface and the y axis. As specified, if the shell has a positive or negative Gaussian curvature, the value of parameter φ will continuously increase or decrease, respectively, and the position of an arbitrary point within the shell will be known by coordinates φ $(0 \le \varphi \le \pi)$ in the meridional direction, θ $(0 \le \theta \le 2\pi)$ in the circumferential direction and ζ $(-h/2 \le \zeta \le h/2)$ along the normal to the mid-surface. But, if the sign of the Gaussian curvature varies along the meridian, coordinate φ should be replaced by a new coordinate