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## Free vibrations of composite oval and elliptic cylinders by the generalized differential quadrature method

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## ABSTRACT

We use the generalized differential quadrature method (GDQ) and shell theories of different order to study free vibrations of laminated cylinders of oval and elliptic cross-sections. In the GDQ method partial derivatives of a function at a point are expressed as weighted sums of values of the function at several neighboring points. Thus, strong forms of equations of motion are analyzed. It is found that the computed frequencies rapidly converge with an increase in the number of grid points along the oval or elliptic circumference defining the cross-section of the mid-surface of the cylinder. For a clamped-free elliptic cylinder the converged frequencies match well with the corresponding experimental ones available in the literature. Furthermore, the lowest ten frequencies computed with either an equivalent single layer theory or a layer wise theory of first order and using shear correction factor are accurate.

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## 1. Introduction

Laminated curved structures are known to be stronger than the corresponding flat ones due to the curvature effect that couples membrane stretching and bending deformations, and the material anisotropy that can be exploited to provide additional strength in the desired directions. In the absence of closed-form solutions for a general doubly-curved structure subjected to an arbitrary load, it is difficult to quantify contributions from these two effects. Books on shell theories and analyses of their deformations include the following: Timoshenko and Woinowsky-Krieger [1], Lekhnitskii et al. [2], Naghdi [3], Leissa [4], Szilard [5], Flügge [6], Gol'denveizer [7], Novozilov [8], Ambartsumyan [9], Kraus [10], Markuš [11], Ventsel and Krauthammer [12], Qatu [13], Carrera et al. [14], Tornabene [15], and Tornabene and Fantuzzi [16]. Rather than making kinematic assumptions (i.e., assuming a displacement field) for a shell Cosserat and Cosserat [17] regarded a shell as a surface with a director (3-dimensional vector) attached to each point of the surface. Deformations of the director account for transverse shear and transverse normal deformations of the shell. Ericksen and Truesdell [18], amongst others, developed kinematics of rods, plates and shells for the Cosserat continuum.

There are numerous studies related to composite shells, and we

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cite a few. Brischetto [19–21] and Brischetto and Torre [22] focused their research on three-dimensional (3D) free vibrations of shells using an exact elasticity solution. Dozio [23], and Hossein and Dozio [24] analyzed laminated composite plates and shells using 2D semi-analytical and numerical approaches. Sofiyev [25], and Sofiyev and Kuruoglu [26] studied transient and buckling problems of functionally graded conical shells. Other related problems for laminated composite and smart materials can be found in [27–37].

We now briefly review some works on vibrations of cylindrical shells of elliptic cross-section. Culberson and Boyd [38] studied free vibrations of simply supported oval (elliptic) cylindrical shells. Sewall and Pusey [39] compared experimental and computed frequencies of circular and elliptic cylinders. They used Sander's shell theory, the Rayleigh–Ritz method, employed cosine and sine series in the circumferential direction and mode shapes of a beam along the axial direction, and found that the number of terms in the circumferential direction needed to get converged values of frequencies considerably increased with an increase in the eccentricity of the elliptic cross-section. They found a reasonable agreement between the computed and the experimental frequencies for clamped-free cylindrical shells. Shirakawa and Morita [40] studied the effect of eccentricity of the cross-section on the natural frequencies and buckling modes of elliptic cylinders. Similar studies were carried out by Armenakas and Koumoussis [41] for the vibration of simply-supported elliptic cylinders. Soldatos [42] presented natural frequencies of anisotropic laminated elliptic

cylinders using the Flügge shell theory. The effect of axial imperfections on vibrations of oval cylinders has been delineated by Hui and Du [43]. Parametric studies related to free vibrations of isotropic and composite structures with variable thickness and cross-sectional Bezier shape have been discussed by Kumar and Singh [44–46]. Recently, Patel et al. [47] considered elliptic cylinders made of functionally graded materials. Hayek and Boisvert [48] studied the same structures using higher-order shell theories, and Ahmed [49] investigated analogous problems using simplified theoretical models. Finally, Sofiyev and Kuruoglu [50] presented the combined influences of shear deformation, rotary inertia and heterogeneity on the frequencies of cross-ply laminated orthotropic cylindrical shells. The papers cited above do not form a complete list of works on elliptic structures. Here we use several higher-order equivalent single layer (ESL) and layerwise (LW) theories to study free vibrations of isotropic, orthotropic, and laminated thin as well as thick elliptic and oval cylinders and compare our results with those obtained by analyzing the corresponding 2-D (a linear shell theory with 6 degrees of freedom per node) and 3-D linear elastic problems by the finite element method (FEM) using Abaqus. We note that in the FEM a weak or variational formulation of equations governing deformations of a shell are derived and values of integrals over an element are approximated by using a quadrature rule. However, in the Differential Quadrature Method (DQM) [51] a numerical solution of the strong formulation is sought. Finlayson and Scriven [52], Gottlieb and Orszag [53] and Boyd [54] have stated that the DQM can be

seen as a weighted residual method. Several issues related to the choice of collocation points and basis functions in the DQM have been addressed by Quan and Chang [55,56]. Shu and Richards [57,58] have introduced the Generalized Differential Quadrature method (GDQM) that is reviewed in [59]. The DQM has been used to study structural problems by Bert et al. [60–64], Civalek [65,66], Tornabene and Viola [67–71], Tornabene et al. [72–76], Ferreira et al. [77,78] and Viola et al. [79–83]. The book by Zong and Zhang [84] describes the DQM and its applications to analyze various problems. One could analyze initial-boundary-value problems by using the strong form finite elements based on DQM [85–95] and the localized version of the DQM [96–97]. Here we use several higher-order shell theories [98–108] and the DQM [109–117] to study free vibrations of oval and elliptic cylinders. It is shown that the computed frequencies of the first ten modes of vibration agree well with those obtained by using the 3D FEM. The first-order ESL and the first-order LW shell theories give reasonably accurate values of frequencies of the first ten modes of vibrations when shear correction factor of  $\kappa = 1/\chi = 5/6$  is used, the higher-order ESL and LW theories provide accurate values of frequencies without using the shear correction factor.

2. shell equations

We use both an Equivalent Single Layer (ESL) theory and a Layer-Wise (LW) theory [98–107] to analyze free vibrations of oval

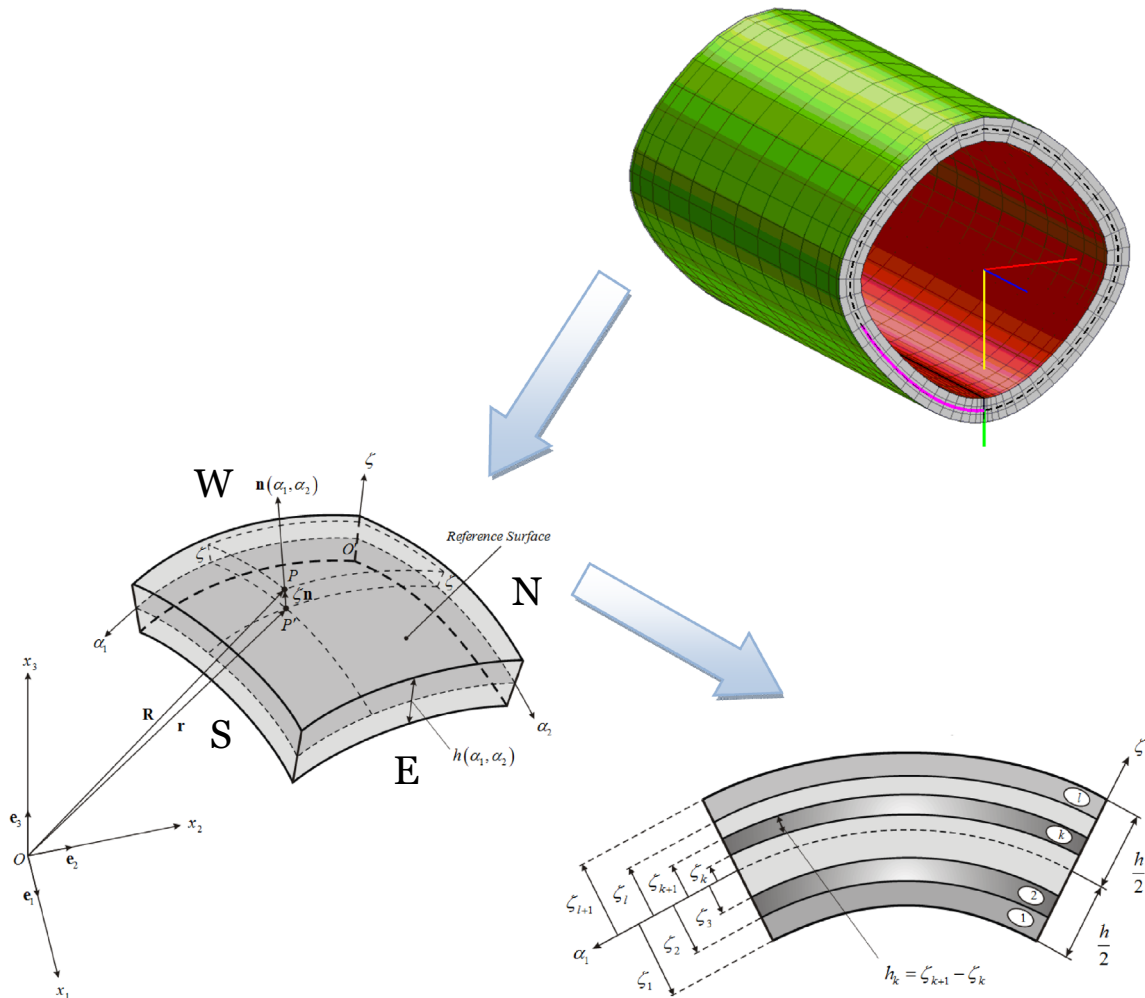


Fig. 1. Elliptic cylinder, fundamental shell element and lamination stacking sequence sketch.

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