Thin-Walled Structures 97 (2015) 207–214

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

# Forced vibration analysis of composite cylindrical shells using spline finite strip method

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ARTICLE INFO

*Article history:*
Received 24 February 2015
Received in revised form
25 August 2015
Accepted 17 September 2015
Available online 30 September 2015

*Keywords:*
Spline finite strip
Forced vibration
Thin composite shell
Fast Fourier Transform

ABSTRACT

In this study, forced vibration behavior of thin-walled composite circular cylindrical shells is investigated using the spline finite strip method (spline FSM). Spline FSM is one of the versions of finite element method (FEM) employing a special element called finite strip. The shells considered in this study are assumed to be thin; therefore, the classical bending theory of shells and Sanders–Koiter's strain–displacement relation are used in the theoretical formulation of developed method. Time-history response of shells concerning arbitrary type of forces, boundary conditions and damping effects are obtained using developed spline FSM. To check the validity of results a comparison has been performed with the results obtained by conventional finite element method using ANSYS software. As far as the time history response of cylindrical shells is available the natural frequencies are extracted using Fast Fourier Transform (FFT) approach. The evaluated natural frequencies of composite shells are then compared with those obtained from an eigenvalue analysis. The comparison of results revealed the fact that developed spline FSM is an accurate tool that can be used in transient and harmonic analyses of composite laminated cylindrical shells.



## 1. Introduction

Thin walled laminated composite shells are fundamental elements in aerospace, marine and civil structures. A comprehensive knowledge about the dynamic response of shells is an essential requirement for design of these structures. In this regard, in the past couple of decades various analytical as well as numerical approaches have been developed by researchers. One of the most efficient methods in dynamic analysis of shell structures is the finite element method (FEM); however, in preliminary stage of design where a huge amount of calculations should be performed for sizing the structural elements the application of conventional forms of FEM may be extravagant and time consuming. Therefore, other versions of FEM like Finite Strip Method (FSM) have been developed. FSM may be considered as a fast tool for structural analysis. Different variants of the method have been developed, namely, semi-analytical FSM and spline finite strip method (spline FSM). Spline FSM was introduced by Cheung [1]. Lau and Hancock [2] used the spline FSM for buckling analysis of thin-walled prismatic structures subjected to longitudinal compression, bending and transverse compression. Also they investigated inelastic buckling of structure with non-linear material stress–strain properties, strain hardening and residual stresses [3]. Cheung and Tham [4] implemented the spline FSM for free vibration analysis of singly curved shell. Dawe and Wang [5] obtained the buckling stresses and natural frequency response of rectangular plates with arbitrary lay-ups and different boundary conditions. They applied the first order shear deformation theory as well as the classical bending theory of plates in the analysis and compared the obtained results with other numerical methods. The comparison of results revealed that spline FSM has good convergence and accuracy properties. Au and Cheung [6] investigated the buckling and free vibration behavior of shells with different geometries using isoparametric spline FSM. Spline FSM is applied to the static and free vibration analysis of piezo-laminated plates, with arbitrary shape and lay-ups, loading and boundary conditions by Loja, Barbosa and Soares [7]. Spline FSM with variable knot spacing was used for calculation of buckling stresses and natural frequencies for multi-span structures by Dawe [8]. Akhras and Li [9] modeled thick piezoelectric composite plates using spline FSM with higher-order shear deformation theory and studied the stability and free vibration of mentioned plates. They investigated the effects of length-to-thickness ratio, fiber orientation, boundary conditions and electrical conditions on the natural frequency and critical buckling load of plates. Fazilati and Ovesy [10] implemented a semi-analytical FSM and spline FSM to calculate the stability and instability regions of flat and curved thin-walled plates under harmonic load. Results demonstrated good accuracy and versatility of the spline FSM. In another research, Ovesy and Fazilati [11] studied

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http://dx.doi.org/10.1016/j.tws.2015.09.014
0263-8231/

the parametric instability regions of laminated composite plate and cylindrical shells subjected to non-uniform in-plane axial end-loadings. They assumed the static as well as varying parts of the end-loading vary according to parabolic distribution in the width of the panel. The dynamic instability of panels has been investigated by using spline finite strip method.

According to authors literature survey the forced vibration analysis of composite cylindrical shells using spline FSM has not been presented in the open literatures yet. Current paper has fulfilled the task and the forced vibration of composite cylindrical shells has been investigated using spline FSM. In this study, effects of different loading conditions as well as different boundary conditions have been taken into account. Moreover the effects of damping on time history response of laminated cylindrical shells have been studied. The natural frequencies are obtained from the time history responses of panels by implementing a Fast Fourier Transform (FFT) scheme.

## 2. Theoretical developments

### 2.1. Theory of splines with equally spaced knots and the related series

The spline function is an estimating continuous-piecewise defined function. It is represented in the form of $B_1$- $B_2$- $B_3$- $B_4$-spline and $B_5$-spline. $B_3$-spline function which is presented as Eq. (1) is used in current study. Moreover, this function is depicted in Fig. (1a).

$$\varphi_m = \frac{1}{6}\begin{cases} 0 & \xi \le -2 \\ (\xi+2)^3 & -2 \le \xi \le -1 \\ (\xi+2)^3 - 4(\xi+1)^3 & -1 \le \xi \le 0 \\ (2-\xi)^3 - 4(1-\xi)^3 & 0 \le \xi \le 1 \\ (2-\xi)^3 & 1 \le \xi \le 2 \\ 0 & \xi \ge 2 \end{cases} \qquad \xi = \frac{x - x_m}{d} \tag{1}$$

In above equation $x_m$ is the coordinate of the center of local spline function. To interpolate the general function $f(x)$ in the interval of $a \le x \le b$ using the spline function, this interval is divided into equal pieces where the distance between any two nodes is $d = (a-b)/q$. As the result, total $q+1$ nodes will be obtained. Corresponding to each node, a spline function is assumed and the spline series are finally obtained as shown in Fig. (1b). In order to apply essential boundary conditions in $B_3$-spline function, two nodes are added to the beginning and end of abovementioned domain.

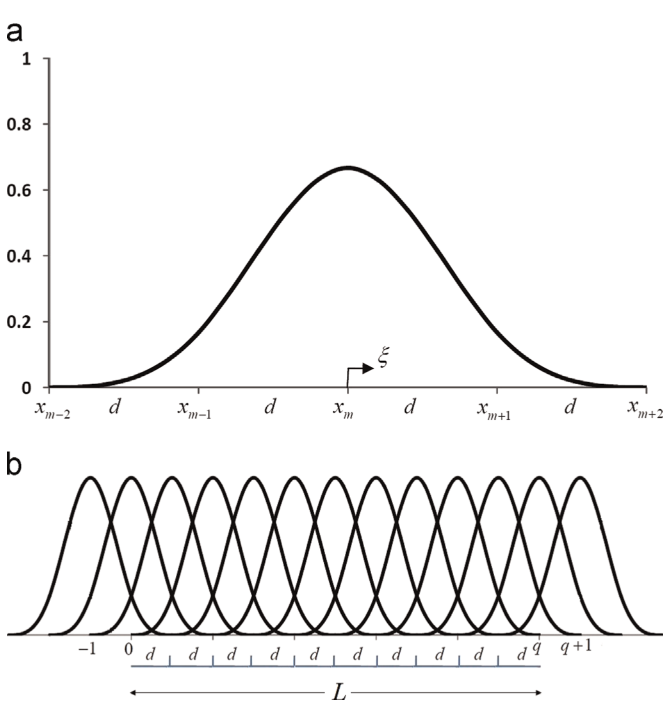


**Fig.1.** (a) Local cubic spline function, (b) $B_3$ Spline series.

Therefore; representation of an arbitrary function $f(x)$ using spline series may be defined as Eq. (2):

$$\begin{aligned} f(x) &= \sum_{m=-1}^{q+1} \varphi_m \alpha_m = [\varphi]\{\alpha\} \\ [\varphi] &= [\,\phi_{-1},\ \varphi_0,\ \varphi_1,\ \ldots\ldots,\ \varphi_{q-1},\ \varphi_q,\ \varphi_{q+1}] \\ \{\alpha\} &= [\alpha_{-1},\ \alpha_0,\ \alpha_1,\ \ldots\ldots,\ \alpha_{q-1},\ \alpha_q,\ \alpha_{q+1}]^T \end{aligned} \tag{2}$$

where, $\alpha_m$ are the unknown coefficients related to each node for obtaining estimating function.

To apply appropriate end boundary conditions, instead of the first two arguments as well as last two arguments of vector $\{\alpha\}$ the function value and the value of first derivative of the function are replaced at the beginning and the end resulting in vector $\{d\}$ as Eq. (3) [12].

$$\{d\} = \left[f(x_0), f'(x_0), \alpha_1, \alpha_2, \ldots, \alpha_{q-1}, f'(x_q), f(x_q)\right] \tag{3}$$

To make a relation between vector $\{\alpha\}$ and vector $\{d\}$ a transformation matrix, $R$ is required.

$$d = R\alpha \quad or \quad \alpha = R^{-1}d \tag{4}$$

The transformation matrix $R$ is represented as following:

$$R = \frac{1}{6}\begin{bmatrix} 1 & 4 & 1 & & & & \\ \frac{-3}{d} & 0 & \frac{3}{d} & & & & \\ & & 6 & & & & \\ & & & \cdots & & & \\ & & & & 6 & & \\ & & & & \frac{-3}{d} & 0 & \frac{3}{d} \\ & & & & 1 & 4 & 1 \end{bmatrix} \tag{5}$$

Finally, to represent the general function $f(x)$ in Eq. (2) the following relation may be used:

$$f(x) = [\varphi]R^{-1}\{d\} = [\bar{\varphi}]\{d\} \tag{6}$$

Where, $\bar{\varphi}$ is called the modified spline function.

### 2.2. Displacement shape functions

The cylindrical shells considered in this paper are assumed to be thin. It implies the application of the classical bending theory of shells in the formulation of spline FSM. Moreover, the Sanders–Koiter's strain–displacement relation has been used to develop the mid-plane strains of finite strip as represented in Eq. (7). The geometry and coordinate system of a spline finite strip is depicted in Fig. (2).

$$\{\varepsilon_\ell\} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{1}{R}\dfrac{\partial \upsilon}{\partial \theta} + \dfrac{w}{R} \\ \dfrac{1}{R}\dfrac{\partial u}{\partial \theta} + \dfrac{\partial \upsilon}{\partial x} \end{Bmatrix} \quad \{\psi\} = \begin{Bmatrix} -\dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{1}{R^2}\dfrac{\partial \upsilon}{\partial \theta} - \dfrac{1}{R^2}\dfrac{\partial^2 w}{\partial \theta^2} \\ -\dfrac{2}{R}\dfrac{\partial^2 w}{\partial x \partial \theta} + \dfrac{1}{2R}\left(3\dfrac{\partial \upsilon}{\partial x} - \dfrac{1}{R}\dfrac{\partial u}{\partial \theta}\right) \end{Bmatrix} \tag{7}$$

Functions used for postulating the displacements in three