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Accurate discrete modelling of stiffened isotropic and orthotropic rectangular plates

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ABSTRACT

An analytical approach is presented here for simply supported blade-stiffened rectangular plates wherein important non-classical effects such as transverse shear deformation and rotary inertia are carefully accounted for. The analysis differs from full three-dimensional modelling of both the plate and the stiffener in that a plane stress idealization is used to model the kinematics of transverse bending of the stiffener while simple one-dimensional classical models are employed for lateral bending and torsion. Parametric studies are used to highlight the importance of non-classical effects in plate and stiffener kinematics and to finally lead to certain recommendations for accurate modelling of stiffened plates.

1. Introduction

Stiffened plates have found extensive use as highly efficient structural elements in aerospace, automobile, naval and civil engineering industries.

The analysis of plates stiffened by closely spaced stiffeners is usually carried out using a "smeared out" approach [1] wherein the stiffened plate is replaced by an equivalent orthotropic plate of uniform thickness. The discrete nature of the structure is lost in such an approach and erroneous results are obtained if the stiffeners are not closely spaced. As an alternative, in a discrete analysis, the plate and the stiffeners are modelled separately while maintaining continuity of the interface tractions and displacements [2]. However, developing general closed form solutions for such systems becomes difficult and this has resulted in more emphasis being laid on various computer-based approximate and numerical schemes using energy principles [3,4], the finite element method [5,6], the finite strip method [7], the constraint method [8] and BEM [9–11].

It is well known that non-classical effects like shear deformation and rotary inertia play an important role in the analysis of thick plates and that these errors get further compounded in composite plates [12,13] because the transverse shear stiffness is quite small in comparison to the bending stiffness. These effects are expected to be quite significant in stiffened plates as well and have been the focus of attention in several papers [14–19], wherein finite element solutions based on first-order and higher-order shear deformation theories were presented.

Developing benchmark solutions to completely characterize the effects of these non-classical phenomena in stiffened plates would require both the substrate plate and the stiffeners to be rigorously modelled as three-dimensional solids. Unfortunately, such an approach is not amenable to exact analytical solution and resorting to full 3D numerical approaches would entail complex and computationally expensive formulations. Hence, it is important to develop simpler analytical models which obviate the need for a complete three-dimensional analysis of the stiffener and yet accurately capture non-classical effects in stiffened plates.

In this regard, an analytical approach was presented recently [20] for isotropic/orthotropic rectangular plates stiffened by a central flat stiffener. The plate was modelled using equations of 3D elasticity and the stiffener using a plane stress idealization. Attention was confined to flexure of the plate symmetrically about the stiffener, such that the stiffener undergoes simple transverse bending without any lateral/twisting deformation. For static loading cases, the individual contributions of the plate and the stiffener to the total shear deformation of the structure were determined by comparing the rigorous formulation with simplified models based on the classical approach. Vibrational frequencies were also studied, again confining attention to modes symmetric about the stiffener, and the errors of the commonly employed classical approach were highlighted. It was shown that these errors are more significant for stiffened plates than for corresponding unstiffened plates.

The objectives of the current work are as follows:

1. To extend the rigorous approach presented earlier to off-centre stiffener configurations. The lateral bending and twisting of the

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stiffeners are duly accounted for using the classical Euler–Bernoulli hypothesis and Saint Venant's theory of torsion respectively, while bending in the transverse direction is modelled rigorously by adopting a plane stress idealization for the flat stiffener.

2. To present static flexure and transverse vibration results based on this approach for isotropic and orthotropic plates with various stiffener configurations. These results are compared with ones based on the classical hairbrush hypothesis for both plate and stiffener kinematics to ascertain the importance of non-classical effects in stiffened plate analysis.
3. To develop a set of useful recommendations for accurate modelling of blade-stiffened plates.

2. Formulation

Consider a rectangular plate of sides a, b and uniform thickness h (Fig. 1) with simply supported edges of the shear-diaphragm type. The plate is integrally stiffened by a single eccentric (i.e. one-sided) rectangular beam of height H and breadth B, attached to, say, the bottom surface of the plate along a line parallel to the x-axis at $y = b^*$. The ends of the stiffener are also taken to be simply supported.

As the plate bends in the transverse z-direction, the stiffener undergoes bending not only in the vertical plane, but also laterally (in the y-direction); it is also twisted along its length. The fundamental assumption of the approach adopted herein is that the bending of the stiffened plate can be analyzed accurately by using a 3D elasticity formulation for the plate so as to capture its transverse shear deformation and thickness stretch effects completely, while using a 2D plane stress formulation for the stiffener in the vertical (x–z) plane so as to capture its vertical shear deformation and transverse normal strain (ϵ_z) effects completely. The other two deformation modes of the stiffener, namely lateral bending and twisting, are expected to be of minor significance to the overall kinematics and hence the former is modelled using the classical Euler–Bernoulli hypothesis and the latter using the classical Saint Venant free warping torsion theory, appropriate for the open cross-section considered here, for which warping restraint effects are negligible. While considering dynamics of the stiffened plate, the corresponding argument is employed as follows. All the inertia effects are considered for the plate by including the x, y and z direction inertia terms in the three equilibrium equations of 3D elasticity. Similarly both the x and the z direction inertia terms are included in the two equilibrium equations of the plane stress formulation adopted for the deformation of the stiffener in the vertical plane thereby fully accounting for rotary inertia in the vertical x–z plane. For lateral bending in the y-direction, only the corresponding lateral translational inertia is considered in the Euler Bernoulli formulation while rotary and in-plane (x-direction) inertias are neglected. Finally, the torsional inertia terms are included in the torsion formulation.

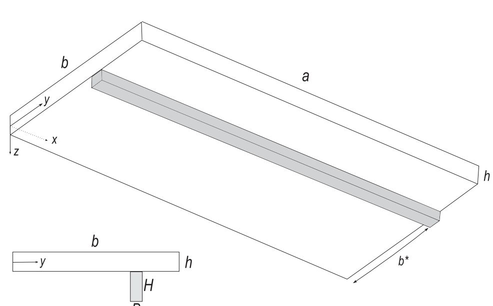

Fig. 1. Eccentrically stiffened plate.

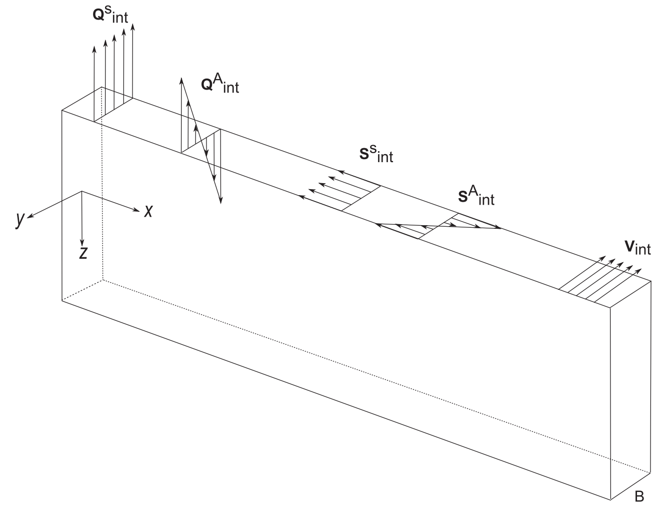

Fig. 2. Interface tractions on the stiffener.

Consistent with the above formulations, a set of appropriate interface tractions has to be introduced with due account of their eccentricity with respect to the centre line of the stiffener. This is explained below in full detail.

2.1. The interface tractions

Because of the eccentricity of placement of the stiffener with respect to the central line $y = b/2$, all three interface tractions need to be considered (Fig. 2) with the possibility of a general variation of the transverse normal and shear tractions across the interface width B. Hence, for the transverse normal and in-plane shear tractions, constant as well as linear antisymmetric variations along the breadth B of the interface patch are assumed while the out-of-plane shear traction is assumed to be constant along the breadth. The mathematical expressions for these tractions, as well as suitable series expansions for later use, are given below:

1. The transverse normal traction $Q_{\text{int}}(x, y, t)$ is taken as

$$Q_{\text{int}} = Q_{\text{int}}^S + Q_{\text{int}}^A = \sum_{m=1}^{\infty} (Q_m^S + Q_m^A \cdot y) \sin\left(\frac{m\pi}{a}x\right) e^{i\omega t} \tag{1}$$

2. The in-plane shear traction $S_{\text{int}}(x, y, t)$ is taken as

$$S_{\text{int}} = S_{\text{int}}^S + S_{\text{int}}^A = \sum_{m=1}^{\infty} (S_m^S + S_m^A \cdot y) \cos\left(\frac{m\pi}{a}x\right) e^{i\omega t} \tag{2}$$

3. The out-of plane shear traction $V_{\text{int}}(x, y, t)$, is taken as

$$V_{\text{int}} = \sum_{m=1}^{\infty} V_m^S \sin\left(\frac{m\pi}{a}x\right) e^{i\omega t} \tag{3}$$

where the superscripts S and A denote the symmetric and antisymmetric part of the tractions respectively and ω is the harmonic frequency, to be taken as zero for the static case.

2.2. Analysis of the stiffener

The plane stress idealization for the stiffener captures the effects of the constant parts of the normal and in-plane shear tractions, Q_{int}^S and S_{int}^S, while the classical Euler Bernoulli beam theory and St. Venant's torsion formulation are used to analyse the lateral bending and torsion of the stiffener due to the remaining traction components. These classical formulations require the tractions to be