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Comparison of theoretical approaches to account for geometrical imperfections of unstiffened isotropic thin walled cylindrical shell structures under axial compression

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ABSTRACT

Thin walled circular cylindrical shell structures are prone to buckle and very sensitive towards geometrical imperfections. The influence of imperfections on the load carrying capacity of shell structures, as they are applied in launcher vehicles is considered by reducing theoretical buckling loads with empirical knock down factors. In general, these knock down factors may lead to a conservative estimate of the load carrying capacity since a worst case scenario is considered. In order to exploit the lightweight design potential of a structure, theoretical approaches to account for geometrical imperfections may lead to more adequate buckling load predictions. Within this contribution different theoretical approaches to account for geometrical imperfections of isotropic shell structures subjected to axial compression are investigated, and the influence of these approaches on the buckling load obtained is studied.

1. Introduction

Unstiffened thin walled shell structures under axial compression are prone to buckle. Within the first half of the last century, a significant deviation between buckling loads determined theoretically and buckling loads determined experimentally was identified. The reason for this discrepancy is explained by the presence of imperfections, i.e. any deviations of the real structure from the perfect shell structure. Thereby, especially the presence of geometrical imperfections is found to have a high degrading effect even though the deviations from the perfect shell structure are within the limits of manufacturing tolerances. In order to account for imperfect shell structures within the preliminary design phase, it is common practice to apply a knock down factor, which is the ratio of experimentally determined buckling loads to theoretical buckling loads of the geometrically perfect shell structure. Several standards of empirical knock down factors have been established, which result from the statistical evaluation of experimentally determined buckling loads.

In Fig. 1, knock down factors commonly applied for isotropic shell structures are plotted against the radius to thickness ratio, R/t. In Eqs. (1)–(5), the mathematical description of these empirical knock down factors established within the second half of the last century is provided. Eq. (1) is developed by Seide [1] in 1964 and recommended in the NASA SP-8007 Space Vehicle Design Criteria [2] published in 1968. Eqs. (2), (3) and (4) are introduced by Almroth [3] in 1974 for different probability levels of 99%, 90% and 50%. These knock down factors are based on the evaluation of broad range of different kinds of shell structures. In Eq. (5), the knock down factor recommended by the European Committee for Standardization (Eurocode 3) is given for steel constructions, which can be adapted to manufacturing quality classes by changing the Q-value accordingly [4]. Therein, a Q value of 16, 25 and 40 represents normal, high and excellent manufacturing quality, respectively.

$$\gamma = 1 - 0.901\left(1 - e^{-1/16(\sqrt{R/t})}\right) \tag{1}$$

$$\rho_{99\%} = 6.48\left(\frac{R}{t}\right)^{-0.54} \tag{2}$$

$$\rho_{90\%} = 8.758\left(\frac{R}{t}\right)^{-0.54} \tag{3}$$

$$\rho_{50\%} = 11.86\left(\frac{R}{t}\right)^{-0.54} \tag{4}$$

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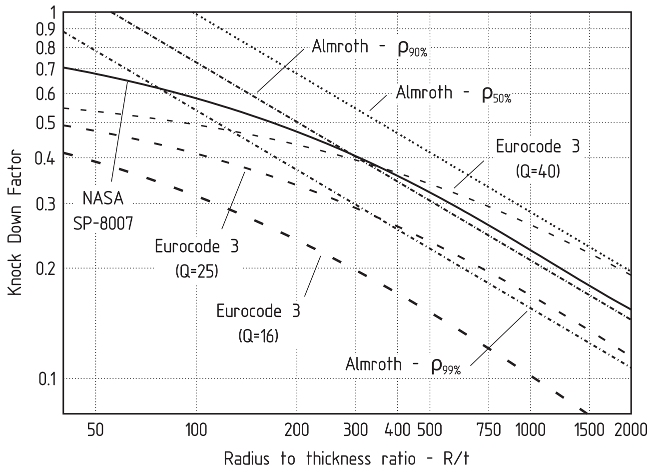

Fig. 1. Comparison of empirical knock down factors.

$$\alpha_x = \frac{0.62}{1+1.91\left(\frac{1}{Q}\frac{R}{t}\right)^{1.44}} \tag{5}$$

These empirical knock down factors are meant to be applied generally for isotropic shell structures to determine a conservative estimate during the early preliminary design phase. In terms of completeness, it has to be mentioned that Takano [5] performed a statistical evaluation of recently performed shell buckling tests of composite cylinders. In contrast to the empirical knock down factors presented above, the knock down factor according to Takano is independent from the $R/t-$ratio and comes to a value of 0.626 for $R/t<544$.

In order to exploit the lightweight design potential of a structure, the application of empirical knock down factors does not mandatorily lead to a proper design since they do not reflect technological advances in manufacturing made within the last few decades. Thus, the application of different theoretical approaches to account for geometrical imperfections may be a suitable alternative in comparison to extensive experimental studies. These approaches allow us to consider the actual imperfection severeness according to the manufacturing process chosen, the geometrical properties, i.e. the radius to thickness ratio, R/t and the length to radius ratio, L/R and boundary conditions of the structure. Thus, the buckling load predictions may appropriately represent the actual physical behaviour of the real shell structure, which may lead to lighter designs.

Within the following progress of this paper, different theoretical approaches to account for geometrical imperfections are presented and evaluated with regard to their physical plausibility. For the latter demand, the question how the imperfection pattern chosen does influence the buckling load is aimed to be answered by applying linear and non-linear computations.

2. Theoretical approaches

Taking real measured imperfections into account by means of transferring them into a suitable structural mechanical model, allows us to predict collapse loads of shell structures theoretically in an accurate manner [6]. Since measured imperfections of real shell structures are not available within the design phase, measured imperfections provided in an imperfection data bank [7] can be applied. In [8], various measured imperfections are evaluated and relations between the imperfection and the manufacturing process are identified. For example, the imperfection pattern of a shell structure made of a number of curved panels assembled to a cylindrical shell structure shows the same number of circumferential full waves within the imperfection pattern as the number of panels used. This type of imperfection is considered as a manufacturing signature [8].

Furthermore, measured imperfections strongly depend on additional influencing parameters, such as the dimensions of the shell. Consequently, the application of real measured imperfections taken from an imperfection data bank is limited to shell structures having similar influencing parameter configurations as those shells being analyzed for the imperfection data bank.

When the knowledge about the imperfection pattern of the shell structure to be designed is limited, further theoretical approaches to account for geometrical imperfections have to be applied. Within this section three approaches for this purpose are presented and applied to isotropic shell structures, subsequently.

2.1. Non-rotational-symmetric imperfections

Since real measured imperfections normally feature both, imperfections in the longitudinal direction as well as in the circumferential direction, non-rotational symmetric imperfections (NRSI) might be considered as simplification of real imperfections. For this purpose, a simplified non-rotational-symmetric imperfection pattern can be represented by a combination of sinusoidal half waves in longitudinal and sinusoidal full waves in the circumferential direction with a certain imperfection magnitude, w.

$$w^*(x,\varphi) = w\ \sin\left(\frac{m_0\pi x}{L}\right)\cos\left(n_0\varphi\right) \tag{6}$$

Mathematically, this kind of imperfection pattern can be described as given in Eq. (6). The variable x and φ are the coordinates in the longitudinal and circumferential direction, respectively.

In Fig. 2, the circumferential imperfection pattern is depicted schematically for an initial number of full-waves $n_0=4$, whereby Fig. 3 illustrates the imperfection pattern in the longitudinal direction with an initial number of half-waves $m_0=5$. The resulting non-rotational-symmetric imperfection pattern can thus be accomplished by combining the imperfection patterns provided in Figs. 2 and 3.

In engineering practice, the buckling mode corresponding to a linear eigenvalue of unstiffened shell structures, which is often described as a chessboard like buckling mode, i.e. a non-rotational-symmetric buckling mode, is applied as initial imperfection when performing finite element computations. For this purpose, single buckling modes or the combination of various buckling modes are used as initial imperfection and the imperfection amplitude can be adjusted by a scaling factor as recently done in [9]. In order to foster the understanding of the influence of the non-rotational symmetric imperfection pattern on the buckling load, the imperfection pattern as described mathematically in Eq. (6) is considered within this contribution.

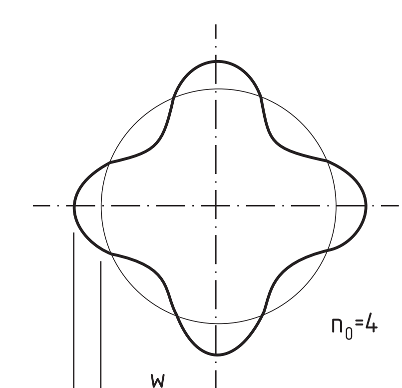

Fig. 2. Non-rotational-symmetric imperfection (NRSI)—imperfection in the circumferential direction $n_0=4$.