Thin-Walled Structures 92 (2015) 65–75

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Frequency analysis of the nonlinear viscoelastic plates subjected to subsonic flow and external loads

Davood Younesian*, Hamed Norouzi

Center of Excellence in Railway Transportation, School of Railway Engineering, Iran University of Science and Technology, Tehran 16846-13114, Iran

ARTICLE INFO

Article history:
Received 24 October 2014
Received in revised form
30 January 2015
Accepted 1 February 2015
Available online 17 March 2015

Keywords:
Von-Kàrmàn plate
Galerkin's approach
Bernoulli's principal
Potential Theory
Multiple Scales method

ABSTRACT

Frequency analysis of the nonlinear viscoelastic plates subjected to the subsonic fluid flow and external loads is presented in this paper. Von-Kàrmàn plate assumptions have been applied and the governing equation of motion of the plate has been derived considering Kelvin's structural damping model. Nono-dimensional forms of the governing equations are derived and the Galerkin's approach is employed to discretize the continuous system. Using Bernoulli's principal, the pressure distribution formula is obtained to model the fluid flow affecting the plate. Multiple Scales method has been used to solve the nonlinear equation of motion. Frequency response curves, time history responses and state space graphs have been obtained for the non-resonance, primary resonance, super-harmonic resonance and sub-harmonic resonance cases. Stability of the solutions has been analyzed and in a parametric study, effects of different parameters on the frequency responses have been studied.

1. Introduction

Plates and shells are widely used as structural members in all over the engineering world. Among all the applications of the plates, their employment in car bodies, high-speed train and airplane structures, make it necessary to analyze the performance of the plates in the fluid flow fields.

Abe et al. [1] used Multiple Scales approach to investigate the sub-harmonic resonance of the simply supported rectangular laminated plates. They used Hamilton's principle to derive the governing equations of motion and applied Galerkin's approach to the equation to obtain Duffing-type nonlinear equation in terms of the transverse displacement. Flow-induced vibrations and hydro-elastic instabilities of rectangular parallel-plate assemblies were studied by Gou and Paidoussis [2]. They employed the extended Galerkin method and Fourier transform technique to solve the plate equation and the perturbation pressure from the potential flow equations respectively. Gou and Paidoussis [3] also analyzed theoretically the linear stability of rectangular plates with free edges in an inviscid channel flow. They employed energy balance analysis to show how different types of instability arise for plates with different supports. The nonlinear aero-elastic behavior of functionally graded plates in supersonic flow was studied by Haddadpour et al. [4]. They used von-Kàrmàn theory in conjunction with the piston theory to model structural nonlinearity and quasi-steady aerodynamic panel loading, respectively. They founded that the use of functionally graded materials significantly changes the flutter behavior of the plate particularly in post-flutter region.

Korbahti and Uzal [5] introduced an analytical solution for the eigenfrequencies of the oscillations of an orthotropic plate placed in a rigid channel of rectangular cross section subjected to fluid flows. They found that, for the most part in case of a composite plate within a duct, the minimum velocity at which the unstable oscillations will occur increases by the employing the strengthening fibers perpendicular to flow direction. Hao et al. [6] performed an analysis on the nonlinear dynamics of a simply supported rectangular plate with functionally graded material (FGMs) subjected to the transverse and in-plane excitations in a thermal environment. Bifurcation and chaotic motion of a thin circular functionally graded plate in thermal environment was studied by Yuda and Zhiqiang [7]. They investigated effects of geometric nonlinearity and temperature-dependent material properties.

Li et al. [8] used Melnikov's method to investigate chaotic behavior of a two dimensional thin panel in subsonic flow. Li et al. [9] also investigated nonlinear dynamical behavior of a two dimensional thin panel with an external excitation and subjected to subsonic flow. They considered the nonlinear cubic stiffness and viscous damper in the middle of the panel and used Potential Theory and Galerkin's method to obtain governing equation of motion. Li et al. [10] studied dynamic behavior of panels exposed to subsonic flow and subjected

* Corresponding author.
E-mail address: younesian@iust.ac.ir (D. Younesian).

http://dx.doi.org/10.1016/j.tws.2015.02.001
0263-8231/

to external excitation based on the von-Kàrmàn's large deflection equations of motion. Their results showed that the panel loses its stability by divergence in subsonic flow and the number of the fixed points and their stabilities change after the dynamic pressure exceeds a critical value. Bifurcation phenomena and scaling properties of a subsonic periodically driven panel with geometric nonlinearity was analyzed by Li et al. [11]. Sort of interesting scaling properties of the bifurcation structure were discussed by them theoretically based on the linear approximation in terms of a discrete mapping. They showed a good agreement between proposed approximate analytical method and the numerically found scaling properties. Tang et al. [12] studied chaos control in a two dimensional panel in the subsonic flow with geometric nonlinearity using Melnikov's function technique. They performed a satisfactory suppression by adding a parametric excitation term into the chaotic system. Yao and Li [13] investigated the bifurcation and chaotic motion of a two-dimensional composite laminated plate with geometric nonlinearity subjected to incompressible subsonic flow and transverse harmonic excitation. Their results showed that the critical divergence velocity of the laminated plate decreases with the increasing ply angle. Stochastic analysis of a nonlinear panel in subsonic flow with random pressure fluctuations was conducted by Li et al. [14]. They showed that a bifurcation of fixed points occurs under a specific condition and the bifurcation point is determined as functions of noise spectral density, dynamic pressure, and the panel structure parameters. Sadri and Younesian [15] examined nonlinear free vibration of a plate-cavity system using Galerkin's method and Harmonic Balance approach. They carried out a parametric study on the system and investigated the effects of different parameters on the value of nonlinear natural frequency. Geometrically non-linear vibrations of a thin infinitely long rectangular plate subjected to axial flow and concentrated harmonic excitation were investigated by Tubaldi et al. [16] for different flow velocities. They showed their results through bifurcation diagrams of the static solutions, frequency-response curves, time histories, and phase-plane diagrams. Li and Yang [17] studied the non-linear dynamical behavior of a cantilevered plate with motion constraints in subsonic flow. They examined the complex non-linear behavior in the region of dynamical instability employing numerical simulations. The region of dynamical instability was divided into four sub-regions by them based on different types of plate motion. Their results showed that symmetric and asymmetric limit cycle motions would occur after dynamical instability. Symmetric and asymmetric period-3 and period-6 motions were observed along with chaotic motions.

Surveying the literature shows that studies have been more focused on finite–infinite types of plate structures. For such an assumption, due to invariance of the geometry and the loading, all the derivatives in transversal direction are vanished. The objective in this paper is to extend the solutions for nonlinear dynamic behavior of definite–definite plates subjected to subsonic flows. Frequency responses are extracted for different resonance conditions including non-resonance, primary resonance, super-harmonic resonance and sub-harmonic resonance circumstances.

2. Mathematical modeling

2.1. Equation of motion of the plate

Fig. 1 illustrates schematic representation of a simply supported plate that is subjected to a subsonic fluid flow and external excitation *f*(*x*,*y*,*t*) simultaneously. The dimensions of the plate and used coordinates have been demonstrated in Fig. 1.

Equation of motion of the plate based on the von-Kármán assumptions and using theory of elasticity can be derived and showed

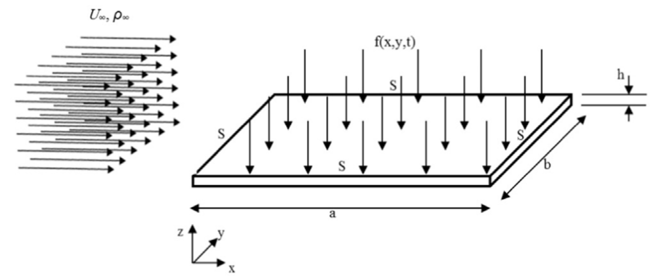

Fig. 1. Schematic representation of the plate subjected to subsonic flow and external excitation.

in the form [18,19]

$$\begin{cases} D\nabla^4 w + ch\dot{w} + \rho h\ddot{w} = f + P + \left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial^2 y}\right) \\ \nabla^4 F = Eh\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \end{cases} \tag{1}$$

In this equation *a*, *b*, *h* and *w*, are the length, width, thickness and transverse displacement of the plate respectively. Also *E*, *D*, ν, *c* and ρ, are modulus of elasticity, rigidity, Poisson's ratio, viscous damping and mass density of the plate respectively, and one can write $\nabla^4 = ((\partial^2/\partial x^2) + (\partial^2/\partial y^2))^2$ and $D = (Eh^3/12(1-\vartheta^2))$. In the above equation, *P* and *f* represent external pressure distribution and other distributed forces separately, and one can add *P* to *f* to obtain total external distributed force. Moreover, *F* is called the potential function and can be found as

$$\frac{\partial^2 F}{\partial x^2} = N_y, \quad \frac{\partial^2 F}{\partial y^2} = N_x, \quad \frac{\partial^2 F}{\partial x\partial y} = -N_{xy} \tag{2}$$

in which N_x and N_y are the normal forces per unit length in the *x* and *y* directions respectively, and N_{xy} is shear force per unit length. Based on the strain–displacement relations, one can write N_x, N_y and N_{xy} as [20]

$$N_x = \frac{Eh}{1-\upsilon^2}\left[\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\upsilon}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]$$

$$N_y = \frac{Eh}{1-\upsilon^2}\left[\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\upsilon}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$N_{xy} = \frac{Eh}{2(1+\upsilon)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right] \tag{3}$$

where, *u* and *v* are in-plane displacements and are equal zero according to von-Kármán theory. Based on this assumptions and after substituting Eq. (3) into Eq. (2) and then substituting the result into Eq. (1) the equation of motion of the plate can be obtained explicitly in the form

$$D(1+g_s(\partial/\partial t))\nabla^4 w + ch\dot{w} + \rho h\ddot{w} = f + P_a + \left[\left(-N_{0x} + \frac{Eh(1+g_s(\partial/\partial t))}{2a(1-\upsilon^2)}\int_0^a \left(\left(\frac{\partial w}{\partial x}\right)^2 + \upsilon\left(\frac{\partial w}{\partial y}\right)^2\right)dx\right)\left(\frac{\partial^2 w}{\partial x^2}\right) + \left(-N_{0y} + \frac{Eh(1+g_s(\partial/\partial t))}{2b(1-\upsilon^2)}\int_0^b \left(\left(\frac{\partial w}{\partial y}\right)^2 + \upsilon\left(\frac{\partial w}{\partial x}\right)^2\right)dy\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + \frac{Eh(1+g_s(\partial/\partial t))}{(1+\upsilon)}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}\right] \tag{4}$$

where, dots (.) denote the derivative with respect to *t*, and g_s is the structural damping coefficient. For the modeling of the viscoelasticity effects of the plate material, one can replace $E \to E(1+g_s(\partial/\partial t))$ [10]. Moreover, the external excitation *f*(*x*,*y*,*t*) can be assumed as a harmonic function in the from $f(x,y,t) = \sin(\pi x/a)\sin(\pi y/b)\cos\omega t$.