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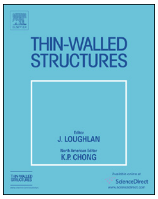

Practical strain-hardening material properties for use in deformation-based structural steel design

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ABSTRACT

Through the development of an innovative full cross-section tensile testing method, a programme of experiments was conducted to investigate the influence of average cross-section properties on the constitutive relationships for carbon steel, to validate the use of an elastic linear hardening model in practical design, and to assess the resulting accuracy enhancements to the new deformation-based continuous strength method (CSM) of structural steel design. A total of seventeen full cross-section tensile tests on hot-rolled I-sections, hollow sections and cold-formed hollow sections were performed and these were compared with coupon test data obtained from a supplementary programme of 14 tensile coupon tests and data carefully obtained from the literature. The overall behavioural response of the cross-section tensile tests demonstrated that assuming an elastic, linear hardening material model for the CSM is a reasonable assumption and the previous assumption concerning the magnitude of the strain-hardening modulus, based upon the recommendations of EN 1993-1-5, is overly conservative. A revised suite of material models was presented and was shown to furnish the CSM capacity equations with a higher degree of accuracy when compared against experimental data.

1. Introduction and key design aspects

Current structural design codes generally represent the material stress–strain characteristics of structural steel by means of an elastic, perfectly plastic model. This leads naturally to the concept of elastic and plastic moment capacities and the process of cross-section classification. Although simple, this treatment can lead to overly conservative designs. A newly proposed, deformation-based approach to structural steel design, referred to herein as the continuous strength method (CSM) [1], represents an alternative treatment to cross-section classification that is based upon a continuous relationship between cross-section slenderness and deformation capacity, as well as a rational exploitation of strain-hardening. Strain-hardening can be broadly defined as the additional strength beyond yield arising as a result of plastic deformation; its importance in the design of steel structures has been previously recognised, notably by [2,3]. The continuous strength method has been shown to offer increases in member resistance of up to 15% over current European standards, as well as a reduction in scatter when compared with test data.

Amongst the key parameters required to develop and use the CSM, material properties are of fundamental importance. For the CSM, two basic assumptions are made: (1) the underlying material model is elastic, linear hardening and (2) in the elastic range the relationship between stress and strain is defined by Young's modulus E and beyond the yield stress f_y this relationship is defined by a strain-hardening modulus, taken as $E_{sh}=E/100$ as recommended by EN 1993-1-5. An elastic, linear hardening model is able to represent strain-hardening effects, and the slope can be adjusted to suit the grade, section type and forming method. Tensile coupon test data typically exhibit a prolonged Lüders or yield plateau with the implication that prior to the onset of any strain-hardening, significant strains must develop; however in this paper it will be demonstrated that this plateau is substantially eroded when considering the stress–strain response of the full cross-section, due to it encompassing variable plate thickness, residual stresses and localised strain-hardening due to cold-forming, as well as variations in the yield stress throughout the cross-section.

Basing structural design equations on full-cross section tests in compression are well documented (see [4]); the purpose of this paper is to take advantage of a modification of this approach, whereby the whole cross-section is tested in tension with the aim to

(i) Examine the strain-hardening behaviour of various hot-rolled and cold-formed carbon steel sections to determine section dependent values of E_{sh}.
(ii) Propose a suite of material models suitable for the CSM as an enhancement to the general provisions made by EN 1993-1-5.

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(iii) Validate the assumption of using an elastic, linear hardening stress strain curve with immediate strain-hardening in the post-yield range.
(iv) Quantify the accuracy enhancements to the CSM resulting from the improved material models.

A wide range of parameters (steel grade, cross-section shape, forming process, loading conditions and local plate thickness) affecting the stress–strain response of structural steel have been identified and their relevance to the CSM has been discussed by Wang [5] whose conclusions were drawn from the analysis of tensile and compressive coupon test data.

Assuming the same general properties (steel grade, cross-section shape, forming process and loading conditions), the stress–strain response of any given specimen will encounter location specific variations in material properties (i.e. the material properties will vary depending on the location from which the coupon is extracted), which are determined by factors such as plate thickness, work hardening due to forming and the distribution of residual stresses due to differential cooling rates through the cross-section. A coupon test will only provide a representative stress–strain response for the area from which it has been cut; coupons taken from multiple locations will provide a family and hence a range of stress–strain responses, but these will still neglect any interactions that develop when the full cross-section is stressed.

1.1. The continuous strength method

The continuous strength method is a deformation-based design approach for steel elements that allows for the beneficial influence of strain-hardening. To date, design equations for the CSM have been developed for cross-section resistance in bending and compression [6]. The CSM bending resistance function $M_{\mathrm{csm,Rd}}$, which applies for $\overline{\lambda}_{\mathrm{p}} \le 0.68$ is defined in (1) as

$$M_{\mathrm{csm,Rd}} = \frac{W_{\mathrm{pl}} f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \left(1 + \frac{E_{\mathrm{sh}}}{E} \frac{W_{\mathrm{el}}}{W_{\mathrm{pl}}} \left(\frac{\epsilon_{\mathrm{csm}}}{\epsilon_{\mathrm{y}}} - 1 \right) - \left(1 - \frac{W_{\mathrm{el}}}{W_{\mathrm{pl}}} \right) \left(\frac{\epsilon_{\mathrm{csm}}}{\epsilon_{\mathrm{y}}} \right)^{-2} \right) \tag{1}$$

where E is the modulus of elasticity, E_{sh} is the strain-hardening slope taken equal to $E/100$ for structural steel, W_{el} and W_{pl} are the elastic and plastic section moduli and $\epsilon_{\mathrm{csm}}/\epsilon_{\mathrm{y}}$ is the strain ratio, defining the limiting strain in the cross-section ϵ_{csm} as a multiple of the yield strain ϵ_{y}, and given by (2)

$$\frac{\epsilon_{\mathrm{csm}}}{\epsilon_{\mathrm{y}}} = \frac{0.25}{\overline{\lambda}_{\mathrm{p}}^{3.6}} \quad \text{but} \quad \le 15 \tag{2}$$

in which $\overline{\lambda}_{\mathrm{p}}$ is the local cross-section slenderness, given by (3) as

$$\overline{\lambda}_{\mathrm{p}} = \sqrt{\left(f_{\mathrm{y}}/\sigma_{\mathrm{cr}} \right)} \tag{3}$$

with σ_{cr} being the elastic buckling stress of the cross-section, or conservatively its most slender constituent plate element.

This research will present (1) a brief summary of the most widely adopted material modelling approaches; (2) a summary of previous studies and proposals for material models based on local coupon test data; (3) the results of an experimental programme carried out at Imperial College London; (4) an updated proposal for the material models to be used in the CSM, taking into account the average tensile cross-section stress–strain properties determined in the experimental investigation.

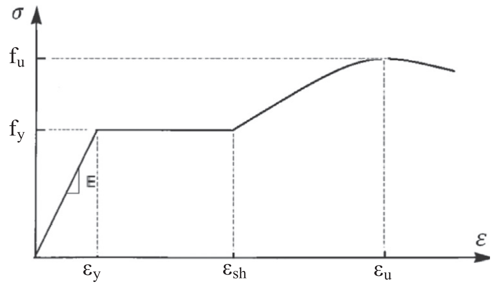

Fig. 1. Typical stress–strain curve for hot-rolled carbon steel.

2. Overview of material modelling approaches

2.1. General

The typical mechanical properties of hot-finished structural steel subjected to static uniaxial load are illustrated Fig. 1. In the elastic range the slope is linear and defined by the modulus of elasticity (Young's modulus) E, which is valued at 210,000 N/mm^2 in EN-1993-1-1. The elastic range is limited by the yield stress, f_y, and corresponding yield strain ϵ_y. Beyond ϵ_y a plateau forms with no increases in stress until ϵ_{sh} is reached, which is the strain at which strain-hardening initiates. At this point, stress accumulation recommences at a reduced rate E_{sh} which is the tangent modulus of the slope at the onset of strain-hardening.

Various idealisations of this relationship exist and can be grouped as (1) elastic or rigid, perfectly plastic; (2) elastic, linear hardening; or (3) elastic, multi-linear hardening or non-linear hardening. The rigid plastic model is illustrated Fig. 2a and forms the basis of current plastic design methods that neglect strain-hardening. Linear hardening models have at least two distinct phases of stress accumulation characterised by the initial slopes at each transition strain. For the elastic, linear hardening model illustrated in Fig. 2b there is an initial elastic phase where stress and strain are related by E, followed by a strain-hardening phase whose rate of stress accumulation is reduced by some proportion of E to give E_{sh}.

A comprehensive review of the form of material model to be adopted by the CSM is given in Wang [5] where it was identified that the following criteria should be satisfied:

(i) A minimal number of parameters.
(ii) Overall accuracy of the stress–strain description for mechanical behaviour.
(iii) Consideration of strain-hardening.
(iv) Stress can be solved for explicitly.
(v) Consistency with the current design code (Eurocode 3).

It was concluded that the elastic, linear hardening model best satisfied the criteria and has since been used throughout the development of the CSM elsewhere [6].

2.2. Current modelling approaches adopted by the CSM

Early work on the CSM concentrated on applications to stainless steel [7–9], and as such employed the Ramberg–Osgood material model. Subsequent extensions to carbon steel [1] have led to the general application of the elastic, linear hardening material model; this model has recently been applied to stainless steel design [10], motivated by its simplicity and consistency with current design codes.

EN 1995-1-1-5 suggests a value of $E_{sh} = E/100$ for all types and grades of steel section. Previous work conducted by Wang [5] on carbon steel properties demonstrated that such a generalised