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Torsional analysis of multi-cell multi-tapered composite beams with cantilever configuration

Sohail Ahmed [a,*], M.N. Ahmed [b]

[a] National University of Science & Technology, CAE, Islamabad, Pakistan
[b] Institute of Avionics and Aeronautics, Air University, Islamabad, Pakistan

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ABSTRACT

The structural performance of thin walled multi-cell multi-tapered (discrete variation of tapered angles at any point along the beam length) composite beams subjected to constrained torsional loading is examined in this paper. A simplified analytical procedure for determining the constrained torsional response of a particular class of thin walled multi-cell multi-tapered composite beams is explained in some detail. The constrained condition analyzed is that of the cantilevered multi-cell multi-tapered beam with torque applied at the free end of the beam. In order to avoid the elastic couplings between bending, torsion and axial effects in the beams, laminates are layed-up symmetrically about their own mid-planes in such a manner that they possess in-plane orthotropy. The analysis approach essentially makes use of the existing theories of torsion appropriate to non-isotropic nature of composite constructions. The details of the finite element analysis are also included in the paper. The comparisons between the theory and finite element results are shown to give close agreement.

1. Introduction

The torsional response of thin walled multi-cell multi-tapered composite beams which are under constrained loading condition is analyzed in this paper. Under cantilever configuration thin walls of the beam are required to resist axial component in addition to the shear component of the stress as required in the case of free torsion. With regard to the isotropic materials, Von Karman and Christensen [1], Fine and Williams [2], Benscoter [3] and Waldron [4] have made the valuable contributions. The analytical review of Von Karman and Christensen [1] and Fine and Williams [2], though noticeably different in their approach, use the common assumption of ignoring the effects of the warping shear strains on torsional response and only employ the St. Venant shear strain in their analysis. The effect of the warping shearing strains on beam deflections is implemented in the works of Benscoter [3] and Waldron [4]. Results of ignoring the effects of the warping shear strains by Von Karman and Christensen [1] on single cell box beams were demonstrated in the research of Loughlan and Ata [5–7].

Much of the research regarding composite open or closed structural sections, has been associated with the effect of global stiffness couplings on structural response [8–10]. Contour analysis for aero-elastically tailored composite rotor blades has been presented by Mansfield and Sobey [8]. Chandra [9] calculated static displacements of a composite box beam and compared the analytical, experimental and finite element results. Chandra and Chopra [10] studied theoretical and experimental behavior of composite I-beams with elastic couplings and showed that the overall bending–torsion coupling of the I-beams significantly influenced by the local extension–twist coupling of the individual flange elements of the sections.

Loughlan and Ata [5–7,11] have studied the constrained torsional response of open and single-cell closed-section carbon-fiber composite beams using a simplified engineering analysis procedure which essentially makes use of the existing isotropic theories of torsion suitably modified to account for the non-isotropic nature of typical carbon-fiber composite material. Comparisons between theory and finite element and between theory and experiment are given in these works and these are shown to give close agreement. Ahmed [12] extended the approach presented by Loughlan and Ata [5–7,11] for multi-cell prismatic and tapered composite beams. Loughlan and Ahmed [13] presented the details regarding multi-cell prismatic beams. The details regarding multi-cell tapered composite beams are presented by Ahmed and Zahid [14], where as it has no idea regarding the issue that what all the effects of multi-tapering would be on the constrained torsional analysis of the beam and/or whether that theory would be implemented on multi-tapered beams or not, in the present study its has been shown that same theory can be applied to the multi-tapered beams after some appropriate modifications, Many of the

* Corresponding author.
E-mail address: sohail.ahmed.rao@cae.nust.edu.pk (S. Ahmed).

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combinations of the geometrical, material and loading parameters would be used to give the best possible configuration in order to fulfill any design criteria these parameters are given as follows:

(a) Multi-tapered angles.
(b) Point of discrete variation along beam length.
(c) Cross section dimensions.
(d) Length of the beam.
(e) Layup material.
(f) Layup stacking sequence.
(g) Number of lay-ups.
(h) Torque applied.

The beams considered are of cantilever configuration having beam length '*L*' with torque '*T*' applied at the free end and '*z*' measured along the beam from the constrained end. The beams considered are multi-tapered from both sides having tapered angles α_a and β_a up-till beam length 'L_1' and then changed to tapered angles α_b and β_b for remaining part of the beam length. The details of tapered angles α_a, β_a, α_b and β_b are shown in Fig. 1 below. The cell area at restrained end is b^*d, where as the reduced cell area due to tapered angles at any location '*z*' along the beam length is $b'{}^*d'$ if $z < L_1$ and $b''{}^*d''$ if $z > L_1$.

2. Theoretical analysis

The warping displacement distribution *w*, normal stress due to constrained warping σ_r and the warping shear flow q_r, for the case of a cantilevered beam of total length *L* with tapered angles α_a and β_a till beam length L_1 (*where* $L_1 < L$) and changed to tapered angles α_b and β_b for remaining part of the beam length, torque *T* applied at the free end and *z* measured along the beam from the constrained end, are detailed below, complete details regarding development and background of these equations were given by Ahmed [11] and by Loughlan and Ata [12].

$$w = -w(s)^*\theta \tag{1}$$

$$\sigma_\tau = \left[\begin{matrix}\left\{\dfrac{d}{dz}\left(\dfrac{D}{C}\right) + C_1\dfrac{d}{dz}(r_1z)^*e^{r_1z} + C_2\dfrac{d}{dz}(r_2z)^*e^{r_2z}\right\}^*w(s) \\ +\left\{\dfrac{D}{C} + C_1e^{r_1z} + C_2e^{r_2z}\right\}^*\dfrac{d}{dz}(w(s))\end{matrix}\right]^*E^a_{x_{eff}} \tag{2}$$

$$q_\tau = E^a_{x_{eff}}{}^*S_W(s)^*\left[\begin{matrix}\dfrac{d^2}{dz^2}\left(\dfrac{D}{C}\right) \\ +C_1{}^*\left\{\left(\dfrac{d}{dz}(r_1z)^*\dfrac{d}{dz}(r_1z)^*e^{r_1z}\right)\right. \\ \left.+\left(e^{r_1z}{}^*\dfrac{d^2}{dz^2}(r_1z)\right)\right\} \\ +C_2{}^*\left\{\left(\dfrac{d}{dz}(r_2z)^*\dfrac{d}{dz}(r_2z)^*e^{r_2z}\right)\right. \\ \left.+(e^{r_2z}{}^*\dfrac{d^2}{dz^2}(r_2z))\right\}\end{matrix}\right] \tag{3}$$

They have been determined on the basis of open-section theory, however the form of Eqs. (1)–(3) can be applied to single- and multi-cell closed construction when used in conjunction, of course, with the appropriate sectorial properties and other section constants. The analysis approach considered in this paper for the multi-cell multi tapered composite beams assumes the absence of cross-sectorial distortion and this can be achieved in practice through the use of rigid diaphragms equally spaced along the length of the beam. In Eqs. (1)–(3), the terms $w(s)$ and $S_w(s)$ are the sectorial coordinate distribution and sectorial shear function, respectively, for the multi-cell construction. The coordinate distribution $w(s)$ represents the level of the longitudinal out-of-plane warping at any location (*s*) around the cross section profile corresponding to a unit rate of twist of the cross-section and this must, of course, satisfy the locations of zero warping on the cross-section which may be clearly evident from conditions of geometrical symmetry.

The forms of Eqs. (1)–(3) are applicable for single/multi-cell and prismatic/tapered/multi-tapered composite beams, when used in combination with the section and sectorial properties for that specific geometry. In order to calculate the warping displacement '*w*' for the multi cell multi-tapered composite beam, we need to know $w(s)$ and θ. The procedure to calculate the $w(s)$ for prismatic multi-cell beam is given in details by Loughlan and Ahmed [13], same procedure has been used to calculate the $w(s)$ for multi-cell multi-tapered composite beams. The only difference in multi-tapered beams would be the change in tapered angles at any point along the beam length and hence change the rate of continuous reduction in cross section dimensions along the beam length, being maximum at restrained end and minimum at free end. The beams dimensions at any cross section along the beam length can be calculated by $b' = b - 2z\tan\beta_a$, $d' = d - 2z\tan\alpha_a$ for $z < L_1$ and $b'' = b - 2\{L_1\tan\beta_a - (z - L_1)\tan\beta_b\}$, $d'' = d - 2\{L_1\tan\alpha_a - (z - L_1)\tan\alpha_b\}$ for $L_1 < z < L$, where *b* and *d* are the flange and web dimensions at restrained end respectively and b' and d' are the flange and web dimensions at any location *z* (for $z < L_1$) and b'' and d'' are the flange and web dimensions at any location *z* (for $L_1 < z < L$) along the beam length.

The following differential equation Eq. (4) can be used for analyzing torsional response (θ) of single-cell and multi-cell composite prismatic/tapered/multitapered beams.

$$\frac{d\left[E'C\frac{d\theta}{dz}\right]}{dz} - GJ\theta = -T + \frac{d\left[E'C\frac{d\theta_0}{dz}\right]}{dz} \tag{4}$$

The Eq. (4) can be written in the general form given below as Eq. (5) at any location 'z' along beam length.

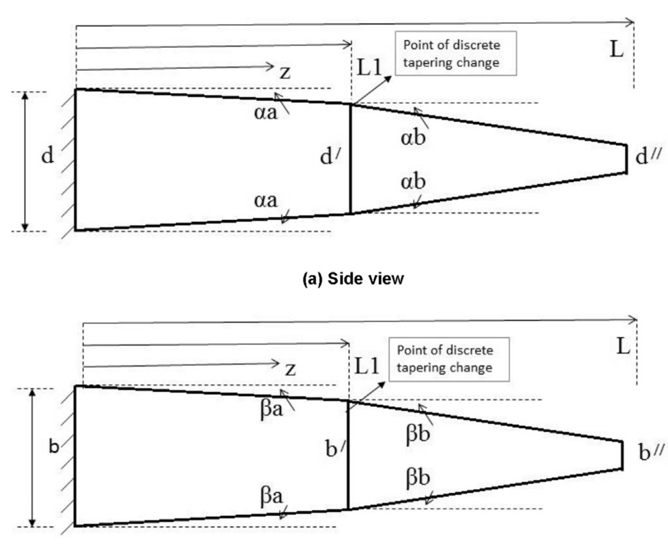

Fig. 1. Description of tapered angles α and β.