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Localised geometric imperfection analysis and modelling using the wavelet transform

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ABSTRACT

Typical methods of analysis and modelling of measured geometric imperfections make use of the Fourier transform to represent the imperfections as summations of sinusoids. The reconstruction of the imperfection usually consists of only a few harmonics, chosen with half-wavelengths equal to the half-wavelengths of the critical elastic buckling modes. This modelling of geometric imperfections is well suited for slender prismatic structures which buckle elastically and for which the buckling modes are harmonic in the longitudinal direction. However, when buckling occurs inelastically, buckling deformations are frequently confined to a relatively small part of the structure and associated with the formation of a spatial plastic mechanism. For this case, the accurate reconstruction of localised imperfections using sinusoids requires the summation of a wide range of frequencies. Further, the Fourier transform cannot explicitly give information on the positioning of localised components. This paper explores the application of the wavelet transform to the analysis and modelling of localised geometric imperfections. Seeing the similarity between various wavelets and the longitudinal variation of typical inelastic buckles, wavelet transforms are expected to be more suitable for modelling the geometric imperfections of non-slender thin-walled structures than the Fourier transform. The theory of the wavelet transform is briefly covered including methods by which practical issues that arise in evaluating the transform, and reconstructing the analysed imperfection, may be resolved. Three imperfection schemes are developed and compared to numerical models incorporating detailed longitudinal and transverse geometric imperfection measurements. These schemes represent the pertinent imperfection components via Fourier series, via wavelet frames, and via a few empirically selected wavelets. The scheme utilising wavelet frames produces excellent results, as does that utilising Fourier series, although it requires a great number of component wavelets due to their finite length. As such, it is worth noting that, in addition to those covered herein, many more methods of reconstruction utilising wavelets are possible.

1. Introduction

Local buckling arises as an instability phenomenon due to the thinness of the component plates comprising the walls of a member. Thin-walled members are generally prismatic and their local buckling behaviour is associated with a number of regular buckles longitudinally; i.e. elastic local buckling. However, buckling does not occur elastically for all such members, and for a plate element whose critical buckling stress is close to the yield stress of its material, local buckling occurs via the formation of a single buckle in a localised region along the member length. Further, as the local buckling stress enters the inelastic range, the buckling half-wavelength decreases to be less than that of the critical elastic local buckling mode, which is typically of the order of the width of the constituent plate elements [1]. This has been observed in tests on members in compression and bending [2–4] and has a theoretical basis in a modified flow theory [5].

Due to the localised nature of inelastic local buckling, the buckling behaviour is most sensitive to localised imperfections. The sensitivity to such imperfections is particularly pronounced where the buckling stress and yield stress coincide [6]. As such, it is necessary to develop a method by which such imperfections may be accurately assessed and modelled. Many methods for modelling imperfections revolve around use of the Fourier transform, e.g. [7,8]. However, since localised imperfection components are often characterised by abrupt changes in the imperfection surface, such characteristics often lead to errors in reconstruction using Fourier series, particularly if higher frequency components are neglected.

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In light of this, a transform that is capable of extracting localised geometric imperfection components may prove useful. Such a transform is the wavelet transform, which provides an analysis with varying resolution and has applications in a wide range of fields [9]. Previously, the wavelet transform has been utilised in multi-scale characterisation of engineered surfaces [10] and geometric imperfection analysis [11], and wavelets themselves have been utilised to directly generate sample imperfections [12], but the wavelet transform itself has not been utilised for such direct modelling of geometric imperfections. This paper will briefly introduce the continuous wavelet transform, methods of reconstructing a signal from its wavelet transform, and the Mexican hat wavelet, including characteristics of the wavelet transform utilising this wavelet. It will further outline detailed geometric imperfection measurements and subsequent stub columns tests carried out on $200 \times 200 \times 6$ square hollow sections (SHS), and detail numerical modelling of the stub columns, including methods for modelling localised geometric imperfections. A number of the functions used herein are available in Wavelet Toolbox in MATLAB [13]. A number of the figures contained herein were also produced using MATLAB.

2. The wavelet transform

2.1. The continuous wavelet transform

Consider a wavelet $\psi(x)$, which is a function with a mean of zero and finite energy; i.e.,

$$\int_{-\infty}^{\infty} \psi(x)dx = 0 \text{ and } E = \int_{-\infty}^{\infty} |\psi(x)|^2 dx < \infty. \tag{1,2}$$

From this wavelet, which is often referred to as a mother wavelet, a set of wavelets may be generated by dilating and translating the mother wavelet as per Eq. (3),

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{x-b}{a}\right). \tag{3}$$

The effect of the scale a is to dilate the wavelet, whereas the effect of the translation b is to translate the wavelet along the analysed direction x. The horizontal distance between any two points of a wavelet, which may be considered a measure of the 'width' of the wavelet, is linearly proportional to the wavelet's scale; i.e. scale and frequency (how rapidly the wavelet oscillates) are inversely proportional. Note that the scale is not necessarily unitless, but rather has the same dimension as the analysed signal (e.g. length or duration). The factor $1/\sqrt{|a|}$ in Eq. (3) is an energy normalisation factor, such that all wavelets generated from the same mother wavelet will have the same energy (E) as defined by Eq. (2). The wavelet of scale one and translation zero is identical to the mother wavelet.

Using the notation of Eq. (3), the continuous wavelet transform of a signal $f(x)$ (such as an imperfection data line) is defined as,

$$W(a,b) = \int_{-\infty}^{\infty} f(x)\overline{\psi_{a,b}(x)}dx \equiv \langle f, \psi_{a,b} \rangle \equiv f(b) * \overline{\psi_{a,0}(-b)} \tag{4a,b,c}$$

where the bar denotes complex conjugation. The wavelet transform may be interpreted as a series of inner products, as in Eq. (4b), between the analysed signal and the set of dilated and translated wavelets. Thus, it provides a measure of 'fit' between the imperfection and the wavelet at different wavelet scales and different translations of the wavelet along the imperfection. The transform may also be written as a convolution as per Eq. (4c) [14].

To satisfy Eq. (2), a wavelet must either be non-zero on a finite interval or decay to zero as $x \to \mp\infty$. As such, wavelets behave like window functions in that they isolate a portion of a signal for analysis; the whole signal is then covered by translating the wavelet over the signal. This is similar to the use of window functions in the short-time Fourier transform; however, as the wavelet width varies with the scale, the size of the wavelet-window varies during the analysis. Due to this, the wavelet transform provides excellent spatial resolution at high frequencies and excellent frequency resolution at low frequencies. How this is achieved is elucidated in Appendix A for the continuous wavelet transform utilising a particular wavelet, known as the Mexican hat wavelet, which is introduced in Section 2.3.

2.2. The discrete wavelet transform [15]

The scales and translations considered in evaluating the wavelet transform need not be continuous. Consider discretisation of the scales and translations of the wavelet transform such that $a = a_0^m$ and $b = nb_0a_0^m$, where $a_0 > 1$, $b_0 > 0$, and m and n are indices for the discretised scales and translations respectively, with $m, n \in \mathbb{Z}$. This discretisation restricts the analysis to positive scales. The wavelets corresponding to this discretisation are,

$$\psi_{m,n}(x) = a_0^{-m/2} \cdot \psi\left(a_0^{-m}x - nb_0\right) \tag{5}$$

The discrete wavelet transform is then exactly as defined in Eqs. (4a–c) for the continuous wavelet transform, but with $\psi_{a,b}(x)$ replaced by $\psi_{m,n}(x)$. As the only difference between the two transforms is the discretisation of the scales and translations, the discrete wavelet transform is equivalent to sampling the continuous wavelet transform on a space-frequency (or time-frequency) lattice, such as that shown in Fig. 1, which is constructed for $a_0 = 2$. The coordinates shown are the corresponding (m, n) for that point.

At $m = 0$ (corresponding to $a = 1$), every wavelet has a *centre frequency* ω_0; the sinusoid of this frequency is the 'best fit' sinusoid to the wavelet. (Determining the centre frequency is briefly outlined in Section 3.2). As frequency and scale are inversely related, the difference in angular frequency ω between two adjacent points in the frequency direction is,

$$\omega_{m+1} = \omega_m = \frac{\omega_0}{a_0^{m+1}} - \frac{\omega_0}{a_0^m} = \frac{\omega_0}{a_0^m}\left(\frac{1}{a_0} - 1\right) \tag{6}$$

As such, at large scales (i.e. large m), which correspond to low frequencies, the analysed points are closely spaced in the frequency direction, and so the discrete wavelet transform provides good frequency resolution at low frequencies, similar to the continuous wavelet transform. In the translation direction, the difference in translation between two adjacent points is,

$$b_{n+1} - b_n = (n+1)b_0a_0^m - nb_0a_0^m = b_0a_0^m \tag{7}$$

As such, at small scales (i.e. small m), which correspond to high

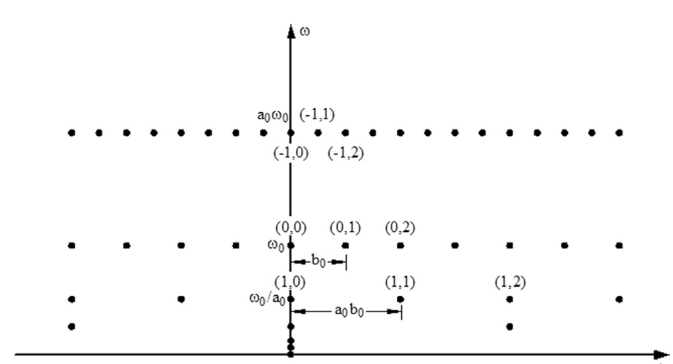

Fig. 1. Time–frequency lattice for the discrete wavelet transform [15].