Thin-Walled Structures 96 (2015) 307–313

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Nonlinear finite element model for the analysis of axisymmetric inflatable beams

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ARTICLE INFO

Article history:
Received 27 April 2015
Received in revised form
21 July 2015
Accepted 24 August 2015

Keywords:
Inflatable Structures
Nonlinear Finite Element
Axisymmetric

ABSTRACT

Inflatable structures are already being used for decades now especially in aerospace applications. The Inflatoplane and inflatable space habitats are just examples. On the other hand, the modeling and simulation techniques of inflatable structures are lacking far behind. Most of the available models are concerned with cylindrical beams. In this paper, a nonlinear Finite Element model for axisymmetric inflatable structures is developed using beam elements. The model is validated by comparing its predictions to two cases of cylindrical beams in the literature. The model is then utilized to predict the effect of two parameters on the wrinkling load of the beam. Results show that the wrinkling load is proportional to the square root of the inflation pressure. For the beam radius, it is proportional to the cube of the radius at small radii but then the relation is linear afterwards. The model is also used to predict the performance of an inflated truncated cone as a function of the inflation pressure and the root radius. The proposed nonlinear Finite Element model is a step towards analyzing real-life inflatable structures.

1. Introduction

Inflatable structures are those structures made of a fabric material and inflated with pressurized gas in order to keep its form. They are capable of withstanding light loads. Inflatable structures are one type of deployable structures which have many uses especially in space applications. They have excellent potential for applications in large sized structures. Solar panels providing power for the Hubble Space Telescope and many satellites, presented by Cawsey [1] are just one application. Large antennas of 20 m diameters and more for communication satellites are another application introduced by Knouse and Weber [2]. They are also used as habitat building. Di Capua et al. [3] designed, developed and tested inflatable habitat elements for NASA Lunar Analogue Studies. Guest [4] reported that these structures were sent to the space using space shuttles which had much smaller diameter. In general, inflatable structures are very efficient for storing in launch vehicles and hence they are very useful in this regard. Also on earth, inflatable structures find many applications. A review of inflatable structures and their applications is presented by Jenkins [5].

A lot of work on inflatable structures was dedicated to the design, development and testing of inflatable wing structures. Some of this work is survey by Cadogan et al. [6]. Goodyear Inflatoplane was developed in the fifties of the last century as a military rescue aircraft. In the seventies, the Apteron unmanned air vehicle with 1.55 m wingspan inflatable wings was developed. Recently, the I2000 unmanned air vehicle was successfully tested by NASA and reported by Murray et al. [7]. A group of researchers and students at the University of Kentucky have developed several versions of inflatable wings in which high-altitude tests are conducted by sending aircrafts with inflatable wings to roughly 30,000 m altitudes on weather balloons to resemble the atmospheric density on Mars as reported by Kearns et al. [8], Usui [9], Usui et al. [10], and Smith et al. [11].

On the other hand, the work dedicated to the modeling and simulation of inflatable structures is lacking behind the efforts in design, building and testing. Fichter [12] presented the nonlinear equilibrium differential equations for the stretching, bending and twisting of pressurized thin-wall *cylindrical* beams. Fichter model was quite difficult to solve due to the high nonlinearities. To avoid this difficulty, he studied two *simple* examples and solved the *linearized* equations to illustrate the applicability of the theory. Breukels and Ockels [13] presented a simple model for the analysis of complex inflatable structures using multi-body dynamics approach. In the last decade, several finite element models for inflatable beams and panels were presented by Thomas and Wielgosz [14], Le Van and Wielgosz [15], Davids [16], Davids and Zhang [17], and Apedo et al. [18]. Recently, Gajbhiye et al. [19] presented a Finite Element analysis model for inflatable torus considering air mass structural element.

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http://dx.doi.org/10.1016/j.tws.2015.08.021
0263-8231/

A large portion of modeling and simulation research on inflatable structures was dedicated to the study of the wrinkling phenomenon. According to Veldman [20], wrinkling is the state in which the sum of the tensile pre-stress and the bending stress has become negative at some location in the beam. The membrane material cannot sustain a negative stress and therefore, it starts to wrinkle. Wrinkling generally occurs first in the location which has the highest compressive stress due to the externally applied load. Main et al. [21] presented a finite element model based on conventional beam theory to calculate the magnitudes of deflection and stress at the onset of wrinkling. Main et al. [22] studied wrinkling based on a strain criterion in another paper. Le Van and Wielgosz [23] studied the wrinkling load based on the linearized Timoshenko beam theory. Recently, Wang et al. [24], and Du et al. [25] studied the wrinkling of membrane inflated cones based on simplified analytical models.

The survey of literature available for inflatable structures shows that there exists a big gap between the inflatable structures applications and the available modeling techniques. Most of the modeling research was directed towards simple cylindrical beams. In this paper, a nonlinear theoretical model is presented, a Finite Element model is developed, validated and utilized to predict the performance of axisymmetric inflatable beams. This paper can be considered as one step towards filling the gap between theory and applications of inflatable structures.

2. Theoretical model

Fig. 1 shows an inflatable axisymmetric beam with a circular cross-section whose length is L and thickness is t. The beam axis is aligned to the x axis. The radius r varies along the axial position such that $r=r(x)$. It is important to mention that by 'axisymmetric' we mean axisymmetric only in geometry such that the cross-section at any point is circular. On the other hand, the loads, displacements, stresses and strains may be axisymmetric or not. The material constants; elasticity and shear moduli (E and G) may be defined independently and hence the definition of these constants holds for orthotropic materials [12].

The point P is located on the beam surface at a distance r from the axis of the beam, and $\mathbf{u}_p$ is the displacement vector of point P including three displacement components $\{u_p \;\; v_p \;\; w_p\}^{\mathbf{T}}$ such that u_p and v_p are tangent to the surface while w_p is normal to it. Assuming small variations of r with respect to x, u_p is in the direction of the beam axis, w_p is along the radius vector and v_p is normal to both of them. The displacement components can be expressed in terms of the corresponding displacement components at the section centroid (along the beam axis) $\{u \;\; v \;\; w\}^{\mathbf{T}}$ as follows

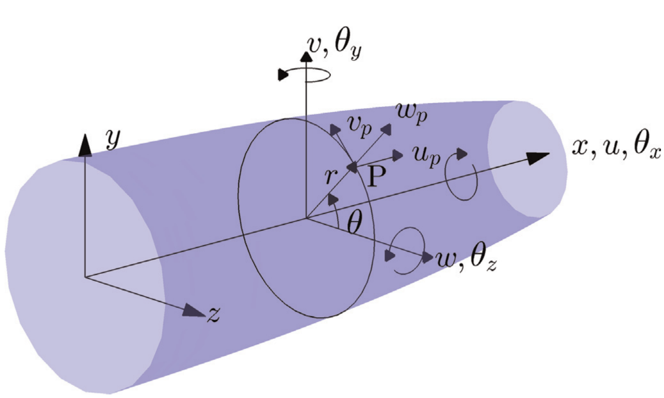

Fig. 1. Schematic of axisymmetric inflatable beam showing displacement components of point P in local coordinates and those in global coordinates at the beam axis.

$$
\begin{aligned}
u_p &= u + r\theta_y \cos\theta - r\theta_z \sin\theta \\
v_p &= v\cos\theta - w\sin\theta - r\theta_x + \frac{r}{4}(\theta_y{}^2 - \theta_z{}^2)\sin 2\theta \\
w_p &= v\sin\theta + w\cos\theta - \frac{r}{2}(\theta_x{}^2 + \theta_y{}^2\cos^2\theta + \theta_z{}^2\sin^2\theta)
\end{aligned}
\tag{1}
$$

where $\{\theta_x \;\; \theta_y \;\; \theta_z\}$ are the angular rotations of the section about the x, y and z axes, while θ is the angular position of the point P as shown in Fig. 1.

According to the principle of virtual work

$$
\delta\prod\nolimits_s + \delta\prod\nolimits_p - \delta W = 0 \tag{2}
$$

where $\delta\prod_s$ is the variation in the strain energy, $\delta\prod_p$ is the variation in the potential energy due to the internal pressure in the inflatable structure, while δW is the work done by external loads.

The variation in the strain energy encompasses the work done by all internal forces (normal forces in x direction N, shear forces in y and z directions V_y and V_z), and the internal moments about x, y and z axes (M_x,M_y, and M_z) and hence the strain energy $\delta\prod_s$ can be expressed in terms of the sum of the products of relevant force and strain components integrated over the domain $0 \le x \le L$, as

$$
\delta\prod\nolimits_s = \int_0^L (\delta\varepsilon_x N + \delta\gamma_y V_y + \delta\gamma_z V_z + \delta\gamma_x M_x + \delta\kappa_y M_y + \delta\kappa_z M_z)dx \tag{3}
$$

where

$$
\begin{aligned}
N &= 2\pi rtE\varepsilon_x \\
V_y &= \pi rtG\gamma_y \\
V_z &= \pi rtG\gamma_z \\
M_x &= 2\pi r^3tG\gamma_x \\
M_y &= \pi r^3tE\kappa_y \\
M_z &= \pi r^3tE\kappa_z
\end{aligned}
\tag{4}
$$

Therefore

$$
\begin{aligned}
\delta\prod\nolimits_s = \int_0^L &\pi rt(2E\delta\varepsilon_x.\,\varepsilon_x + G\delta\gamma_y.\,\gamma_y + G\delta\gamma_z.\,\gamma_z + 2r^2G\delta\gamma_x.\,\gamma_x \\
&+ r^2E\delta\kappa_y.\,\kappa_y + r^2E\delta\kappa_z.\,\kappa_z)dx
\end{aligned}
\tag{5}
$$

The six strain components can be expressed in terms of displacement functions, and their derivatives, up to quadratic terms as derived by Fichter [12] are

$$
\begin{aligned}
\varepsilon_x &= u' + \frac{1}{2}(r^2\theta_x'^2 + v'^2 + w'^2) \\
\kappa_y &= \theta_y' - \theta_x' v' \\
\kappa_z &= \theta_z' - \theta_x' w' \\
\gamma_x &= \theta_x' \\
\gamma_y &= v' - \theta_z + \theta_x w' \\
\gamma_z &= w' + \theta_y - \theta_x v'
\end{aligned}
\tag{6}
$$

Using the strain–displacement relationships in Eq. (6) in Eq. (5) leads to expressing the variation in the strain energy in terms of displacement functions.

The variation in the potential energy due to pressure and volume change $\delta\prod_p$ is