Thin-Walled Structures 95 (2015) 183–195

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

A penalty approach to obtain lower bound buckling loads for imperfection-sensitive shells

Luis A. Godoy [a,b,*], Rossana C. Jaca [c], Eduardo M. Sosa [d], Fernando G. Flores [a,b]

[a] Institute for Advanced Studies in Engineering and Technology, CONICET-UNC, Argentina
[b] Departamento de Estructuras, FCEFyN, Universidad Nacional de Córdoba, P.O. Box 916, Córdoba, Argentina
[c] Civil Engineering Department, Facultad de Ingeniería, Universidad Nacional del Comahue, Buenos Aires 1400, Neuquén, Argentina
[d] Civil and Environmental Engineering Department, West Virginia University, Morgantown, WV 26506-6103, USA

ARTICLE INFO

Article history:
Received 12 June 2015
Received in revised form
3 July 2015
Accepted 5 July 2015
Available online 16 July 2015

Keywords:
Buckling
Finite elements
Reduced energy method
Perturbation methods
Stability
Shells

ABSTRACT

The strategy of Reduced Stiffness (or Reduced Energy) Analysis, in which selected energy components are eliminated to account for mode interaction and imperfection-sensitivity in a simplified way, was developed by Croll and co-workers since the early 1980s. This physical interpretation allows the formulation as an eigenvalue problem, in which the eigenvalue (critical load) is a lower bound to experiments and to nonlinear incremental analysis. This paper considers the computational implementation of both reduced stiffness and reduced energy approaches to the buckling of shell structures by means of perturbation techniques and penalty parameter methods. The structural configurations of interest in this work are cylindrical shells with or without a roof. The reduced stiffness approach has been implemented in a special purpose finite element code for shells of revolution, whereas the reduced energy methodology was implemented in a general purpose finite element code. The present results are compared with geometrically nonlinear analysis including geometric imperfections. Achievements and difficulties in extending the methodologies to complex problems in engineering practice are highlighted.

1. Introduction

Attempts have been made by researchers to develop simple computational tools to provide approximate solutions to shell buckling problems and avoid gross errors in solutions without the need that the user has knowledge of the complete arsenal of shell buckling theory. James G.A. Croll promoted the use of a simple technique based on a reduced version of the energy (or the stiffness) of the shell, in which only an eingen-value problem needs to be solved. This paper discusses ways to implement such methodology in more complex engineering problems using finite element codes.

The European approach to the analysis of shell buckling problems [1] using finite element tools identifies several possible types of analysis, including Geometrically and Material Nonlinear Analysis with Imperfections (GMNIA) as the "best" estimate of buckling capacity; Geometric Nonlinear Analysis with Imperfections (GNIA); Material Nonlinear Analysis (MNA); and Linear Bifurcation Analysis (LBA). An intermediate method is recommended as a "less onerous" approach, which is based on a combination of LBA and MNA. The recommended approach requires design curves that need to be established for each geometric and load configuration, which take the form of elasto-plastic interaction curves. The parameters of such curves should be obtained from a number of full GMNIA, and once the curves are constructed for a specific class they can be used by performing LBA and MNA studies for a given case of interest. The non-specialist engineer who does not have the curves for his/her own problem is therefore lost since the start. A specialist engineer, on the other hand, needs to spend time and effort to develop the tools before using them.

In the American approach the loads are specified, such as in the ASCE provisions [3], but the engineer is left to decide what type of analysis is suited for each case. Of course, this is a job for the specialist engineer, because a novice may mix concepts and approaches to yield incorrect solutions.

The question of what is "onerous" in computational mechanics, as is the concern of the European Committee for shell buckling [1], has considerably changed over the last decades. In 2015 the "onerous" part of the job consists in understanding the physics of the problem and conceptually modeling the case in hand.

---

* Corresponding author.
E-mail addresses: lgodoy@efn.uncor.edu (L.A. Godoy), rossana.jaca@fain.uncoma.edu.ar (R.C. Jaca), eduardo.sosa@mail.wvu.edu (E.M. Sosa), fflores@efn.uncor.edu (F.G. Flores).

http://dx.doi.org/10.1016/j.tws.2015.07.005
0263-8231/

We shall not refer here to shell buckling problems in general terms, but attention will be restricted to very thin shells, with radius to thickness slenderness between 1000 and 2000, such as those employed in the fabrication of storage tanks for the oil industry, which tend to buckle in the elastic range and plasticity develops only in advanced post-buckling states. There are also functional requirements that need to be considered, as in any case in industry: storage tanks usually have an internal floating roof that floats on top of the oil or fuel, and large buckling deflections may be sufficient to block the floating mechanism, with the consequence that the structure has to be left out-of-service until it is repaired.

Regarding the nature of imperfections, one may distinguish between global imperfections (possibly due to errors in fabrication or damage under previous loads) and local imperfections (such as welding defects). Following Koiter's asymptotic formulation [2], in the vicinity of a critical state the most detrimental shape of imperfection is the eigenmode associated with the lowest eigenvalue (this is called an eigenmode-affine imperfection). Because Koiter employed an asymptotic approach centered on a bifurcation point, the validity of his results is limited to small post-buckling displacements and small imperfection amplitudes; this however allows comparisons to be made between different shapes of imperfections. For a number of years, imperfections were almost exclusively assumed in the form of an eigenmode-affine imperfection.

The choice of this shape of imperfection has also been emphasized in the European Recommendations in statements like: "The eigenmodes affine pattern is the critical buckling mode associated with the elastic critical buckling resistance based on an LBA analysis of the perfect shell" ([1], pp. 125).

The research program known as lower bound buckling based on Reduced Stiffness/Reduced Energy Analysis (abbreviated in this paper as RSA and REA, respectively), developed in stages over a period starting in the mid-seventies. A summary of the main contributions is next presented, to highlight the research achievements made during the past 45 years.

The first stage was establishing the physics of the problem and the basis of the methodology. Croll discussed the first account of the lower bound approach, largely based on physical observations on the buckling of imperfection-sensitive shells, as follows: "It is shown that the highly unstable forms of buckling involve essentially a process in which the significant membrane contribution to resistance against incremental displacement at the initial stages of buckling is transferred in the advanced post-buckling states to a situation in which bending energy tends to dominate in providing the resistance to incremental displacements" [4]. This first approach involved some speculation concerning the nonlinear process that occurs at the passage to post-buckling states in cylinders under axial loads.

The first systematic approach for cylinders under lateral pressure was based on analytical studies reported by Batista and Croll. Based on physical reasoning, the authors argued that "the unstable post-critical behavior is the result of the loss of this membrane stiffness" [5]. A simplified methodology was presented in which "appropriate terms in the membrane potential energy are neglected". The results were supported by experiments performed by the authors and were shown to provide a lower bound to experiments of other authors as well. The extension of this methodology to axially loaded cylinders was published a few years later [6].

A second stage was the extension of the methodology to other cylindrical shell configurations, namely stiffened cylinders. Emphasis shifted from understanding the physics of the problem to providing a design methodology, thus addressing more complex shell configurations usually found in off-shore structures and providing simple expressions which could be used in design. Thus, the research program addressed elastic buckling of stringer [7,8] and ring stiffened cylinders [9,10], and combination of ring and stringer stiffeners [11].

Extensions of the lower bound approach to shell configurations other than cylinders were pioneered by Zintillis and Croll in a series of papers on cooling towers under wind or lateral pressure. Following an analysis of the energy components, the reduced stiffness critical spectrum was obtained by suppressing the membrane strain energy U^{2m} from the classical analysis, leading to a reduced critical load

$$\lambda^* = \frac{U^{2b}}{U^{2m} + U^{2b}}\lambda^C \tag{1}$$

where U^{2b} is the bending energy contribution [12]. This computational research was supported by experiments on toroidal and hyperboloidal shells. This was the first analysis performed using a finite element special purpose code to model the shell, and the code was limited to axisymmetric loading. Analysis for combined axial and lateral loading was reported in [13]. For wind-loaded shells, the worst stressed meridian approach was employed thus assuming the equivalence between the asymmetric wind pressure and a symmetric pressure. Uniform thickness was employed in all cases [14]. Pressure-loaded spherical caps were addressed by Goncalves and Croll [15], whereas Kashani and Croll [16] investigated spherical space domes. Other researchers employed the methodology for composite materials [17,18], and this interest has recently been extended to aeronautical applications [19].

A third stage involved the extensions of the methodology to account for elasto-plastic buckling of cylinders. This was done with simple analytical expressions and was reported in Refs. [20–23].

A fourth stage was the computation of nonlinear analyses, which were performed analytically by Yamada and Croll [24–26] using a nonlinear Ritz analysis.

The best readings reviewing the lower bound approach were presented by Croll as a design methodology [27,28] in which details of the motivations and achievements at each stage are discussed in an amenable way.

In summary, the RSA/REA studies by Croll and co-workers were based on

- Analytical methods to obtain explicit expressions for lower bound buckling loads which could be used in design. Use of finite element models was the exception in the work of Zintillis, because explicit expressions could not be found for the configurations of interest.
- Shells with uniform thickness were addressed. Although this may be seen as a trivial simplification in real cases, modeling thickness changes brings some additional difficulties to the application of RSA or REA.
- Shells considered were subjected to axi-symmetric loads (either axial or lateral pressures). Cases of wind-loaded shells were not treated as asymmetric loadings but some form of simplification was used to model axi-symmetric pressures.
- Terms "Reduced Stiffness" and "Reduced Energy" were used indistinctively in the literature. In some cases, although energy expressions were employed, reference to RSA was made.

This paper is concerned with extensions of the methodology to more complex engineering configurations in terms of loads and shell thicknesses, for which finite element analysis is mandatory in order to obtain results. Specifically, results are presented for cantilever cylindrical shells, both with and without a fixed roof.